A STUDY OF THE RELATION BETWEEN SHIP STIFFNESS AND MAXIMUM SLAMMING MOMENTS AMIDSHIPS

Michael Richard Donovan

A STUDY OF THE RELATION BETWEEN SHIP STIFFNESS AND MAXIMUM SLAMMING MOMENTS AMIDSHIPS

by

MICHAEL RICHARD DONOVAN

B.S. Naval Architecture, U.S. Naval Academy

(1974)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE
IN NAVAL ARCHITECTURE AND
MARINE ENGINEERING
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
September 1975

A STUDY OF THE RELATION BETWEEN SHIP STIFFNESS AND MAXIMUM SLAMMING MOMENTS AMIDSHIPS

by

Michael Richard Donovan

Submitted to the Department of Ocean Engineering on August 11, 1975, in partial fulfillment of the requirements for the degree of Master of Science in Naval Architecture and Marine Engineering at the Massachusetts Institute of Technology.

ABSTRACT

The American Bureau of Shipping has recognized the need for a new ship stiffness criteria because of the increasing trend in overall ship dimensions. It has been found that the limiting paramenters which effect hull girder stiffness are slamming, springing, and propeller induced vibrations. This study was done on the overall response of a ship to slamming.

A background on both ship stiffness and slamming is presented. Then the theoretical relationship between slamming and hull response is discussed. An equation relating stiffness to slamming moment amidships is suggested and the appropriate data analyzed to verify the formula. Finally, the use of the relationship in establishing a stiffness criteria is suggested and an example is given.

The data used to verify the stiffness equation was obtained from the Kline-Clough computer program. Slamming moment data for the FOTINI-L, an 800-ft. bulk carrier, the STR. E. L. RYERSON, a 712-ft. Great Lakes ore carrier, and the S. S. MICHIGAN, a 544-ft. general cargo ship were used.

Thesis Supervisor: J. Harvey Evans

Title: Professor of Naval Architecture

ACKNOWLEDGEMENTS

I would like to thank my thesis advisor, Professor J. Harvey Evans, for his guidance and helpful suggestions during the writing of this thesis.

I also want to thank Miss Christine Simony for her encouragement and for her excellent typing.

TABLE OF CONTENTS

TABLE OF FIGURES

Figure | Page

TABLE OF TABLES

INTRODUCTION

An investigation into hull girder stiffness has been initiated by the American Bureau of Shipping (A.B.S.). The ultimate goal of the study is to find a new criteria for ship stiffness and write it in terms of a rule to be used as an A.B.S. standard. Up to the present time, the control of stiffness or deflection has been inherent in classification society rules by limiting the L/D ratio [1].* The following quote is from Section 6,"Longitudinal Strength" of Reference [2],

> ...the equations in this section are, in general, valid for all vessels having depths not less than one-fifteenth of their lengths...

This criteria seems vague and could be overly limiting.

Several other good reasons for looking into hull girder stiffness also exist. For example, there are situations where higher L/D ratios could be beneficial, especially when draft is limited. Also, recently there has been a reduction in section modulus requirements by the various classification societies. This allowed reduction causes new ships to have decreased hull girder inertia thus increasing flexibility.

---

*Numbers in square brackets designate References at end of paper.

Further, the greater use of high strength steel in merchant ships is forcing new study of hull girder flexibility. Finally, the aluminum hull has always been plagued by the deflection problem [2].

To begin with, the Hull Girder Stiffness Criteria Committee carefully considered several aspects of ship stiffness. These aspects included: (1) the triggering effect of hull girder deflection upon instability collapse; (2) the reduction in load-carrying capability from premature immersion of the Plimsoll Mark; (3) slamming response for its contribution of stress components, especially amidships; (4) springing for its cumulative stress effects; and (5) steady state propeller-excited vibratory motions for their deleterious effects upon personnel and main machinery components [3]. After careful study, the Committee eliminated (1) and (2) because of their lesser and even doubtful significance. Thus, (3), (4), and (5) were kept as the primary concerns for the rest of the investigation. From reference [3],

> It is proposed, then, that each of the three primary factors be examined individually, a suitable acceptance criterion sought for each, and a preliminary design formulation derived to define specifically the acceptance limits of hull girder stiffness in each case... Whichever of the three factors was the more demanding would, naturally, be ruling.

The purpose of this thesis is to study the "slam response" factor. That is, to find a relation between slamming moments (and resulting stress amidships) and ship hull girder stiffness. First, a background on both ship stiffness and previous slamming studies will be presented. Then the theoretical relationship between slamming and hull response will be discussed followed by an explanation of the analysis done for this thesis. A relationship of stiffness to slamming moment amidships is suggested and the data analyzed to verify the relationship. Then its application to the hull stiffness criteria study is presented, and finally, conclusions and suggestions for further study are given.

The investigation was accomplished by using a computer program written by R. G. Kline and R. W. Clough. Characteristics for three different ships were also used in the study. These were the FOTINI-L, an 800-ft. bulk carrier, the STR. EDWARD L. RYERSON, a 712-ft. Great Lakes ore carrier and the S. S. MICHIGAN, a 544-ft. general cargo ship. Slamming moment data were obtained for each ship using the Kline-Clough program and then analyzed.

SHIP STIFFNESS

Recently there has been a sudden demand for larger, faster, and different types of ships. These new trends are a result of changes in the economics of marine transportation, the introduction of new types of cargo (containers, LPG, LNG, etc.), the use of high powered machinery, and a wider variety of structural materials [4]. Along with changes in size, form, speed, and type, trends such as lighter scantlings (brought about by improved coatings) and the use of high-strength steel are also coming into play in ship design. The various aspects of these changing trends have directly or indirectly influenced the basic stiffness of the ships' main hull girder, the knowledge of which is vital to many considerations in ship design. Some of these considerations include hull deflection, stress, vibratory response of the primary structure, metal fatigue, human comfort, and loads applied to nonstructural components such as piping and joiner bulkheads. Thus, it has been necessary to start new investigations into different ship structural responses as a function of overall hull girder stiffness. It will be helpful to look into previous studies and observations on ship stiffness.

History of Ship Stiffness Investigations

In the past ten years there has been a demand for the increase in size of bulk carriers and general cargo carriers as well as tankers. In hull stiffness studies of these ships, it has been pointed out that as the size of simple structures increases, the weight becomes proportionately higher. In order to provide equivalent strength, material thickness and weight must be increased above that indicated by geometrical similarity. These increases are because loads and bending moments in a series of geometrically similar hollow beams increase as the fourth power of linear dimensions while the strength increases only as the cube of the linear dimension [1]. As a result, there have been times when it seemed that structural strength and hull stiffness might impose an upper limit on ship dimensions.

At the same time, there has been a reduction in section modulus requirements for tankers and other types of ships. This reduction, which has taken place since 1959, is possible because of the following: (1) greater knowledge of predicted wave sizes; (2) reduction in effective wave heights for use with the conventional static calculation; (3) credit given for all continuous longitudinal material; (4) greater confidence in design concepts by experience with gradual increase in size; and (5) more confidence in new materials to resist brittle

fracture [1]. Also, reduction of corrosion allowances by the use of effective coatings can allow material savings of up to 6% of light ship weight.

The resulting lower section modulus causes a decrease in the transverse section moment of inertia of the hull girder. Assuming no change in the nuetral axis, the reduction in inertia is of the same order of magnitude as the change in section modulus. This means an increase in flexibility and greater hull deflection. While strength aspects of the hull girder have previously been emphasized, some designers are worried that the flexibility could exceed allowable limits, whatever they may be.

As mentioned before, the control of stiffness of a ship has been inherent in classification society rules by limiting the ratio of length to depth. This value must be less than 14 for tankers and bulk carriers and ranges from 12 to 14 for cargo ships. Recently, classification societies have allowed an increase of this ratio to about 16. Freeboard regulations have been based on L/D ratios between 10 and 13.5 for years. Limiting this ratio provided a standard of strength in association with freeboard and undoubtedly some consideration was given to stiffness when standards were established for locating the Plimsoll Mark.

These limited L/D ratios are not acceptable for new larger ships, however. While length and beam have been expanding

in newer ships, draft has been limited by depths of harbor channels, canals, etc. This limited draft, along with freeboard requirements, causes a limit on depth. Thus L/D ratios become larger, making the present stiffness criteria unacceptable.

All rules discussed, so far, have been for medium steel construction. There has been increased use of high strength steel in bulk type carriers, both dry and liquid, and cargo ships with wide hatch openings in the decks. In these instances the result has been a structure of reduced stiffness as compared to previous practice. In most cases it has been flexibility criteria rather than strength which has limited a full application of high strength material [1].

Thus, it can be seen that the larger size of new ships, decreased section modulus requirements, widespread use of high strength steel, and the vague L/D ratio criteria now in use, all motivate new investigations into hull girder stiffness and its effects on the overall ship design.

Importance of Ship Stiffness

The final result of the trends mentioned above is an increase in ship hull flexibility. The general meaning of "flexibility" is well understood; however, it is difficult to derive a precise definition of flexibility which is applicable to ships and acceptable as a basis for comparison of all ship types. This is primarily because of the complexity and non-

uniformity of both the ship's structure and the loading imposed on it. It is also due to the many aspects of structural behavior that are related to flexibility but cannot be adequately described by the same parameters [1].

The term flexibility may be associated with many concepts. It may be described as the inverse or opposite of stiffness. For <u>statically</u> loaded structures, it is related to <u>deflection</u>. That is:

$$\text{deflection} = (\text{load}) \times (\text{flexibility})$$

or

$$\text{load} = (\text{deflection}) \times (\text{stiffness})$$

For <u>dynamically</u> loaded structures it may be related to the <u>natural frequency of vibration</u>. In all structures, it is directly related to strength, but not always synonymous with strength. It is this aspect of flexibility which has caused the greatest confusion [1].

Since all the new design trends are going towards a more flexible ship, it is of interest to know what happens when hull girder flexibility is increased. The following items are the results of increased hull flexibility: (1) lowering of the Plimsoll Mark in the sagging condition with resultant loss in deadweight; (2) change in vibration characteristics; (3) affect on midship stress augmentation from dynamic impulses such as slamming; (4) increase of forces at superstructure and deckhouse connections; (5) deformation and fit of covers over large

openings; and (6) alignment of shafting and deformation of extended systems such as piping [1].

The effect of stiffness on midship stress and bending moments due to slamming is the topic of this thesis and will be discussed further, in detail.

SHIP SLAMMING

Ship slamming refers to the phenomena which occurs when a portion of the hull (usually the bow) impacts the sea surface creating large forces of short duration [4]. Under certain sea conditions, the phase relation between the bow motion and the surface of the oncoming waves is such that these impacts may occur. In most cases it is a result of large pitch and heave motions that force the ship's forward bottom to emerge and reenter the water after hitting its surface. This is known as "bottom impact slamming" [7]. The other type is called "bow flare slamming" and occurs when there is a sudden change in the acceleration of the ship's bow without actual emergence.

There are essentially two problems in dealing with the ship's response to slamming, namely, a localized one, also called a "micro" and an overall or "macro" problem [7]. The "micro" problem deals mainly with local plate forces and damage resulting from direct application of the load. The overall response involves hull vibrations and large midship stresses and bending moments that can be detrimental to the structure as a whole. There has been recent debate over which of these effects is more important.

History of Slamming Studies

Slamming has been called "...one of the most complicated phenomena experienced by a ship operated in rough seas" [9]. In the past forty years, since the slamming phenomenon first came to the attention of naval architects, more than 300 papers on ship slamming have been published with both theoretical and experimental approaches. Authors have tried to establish the necessary conditions required for bottom impact slamming to occur. Some of the experimental approaches used were observations aboard ships at sea, model experiments in waves generated in tow tanks, and water entry drop tests on two-dimensional models [9]. After conducting tests in irregular waves, Ochi came up with two sufficient conditions: (a) bow(forefoot) emergence and (b) a certain magnitude of relative velocity between wave and ship bow [7]. As a result of these and other studies, it has been thought that one of the most important pieces of information needed in association with slamming is the magnitude of the impact pressure. Results obtained from either full-scale trials or seakeeping model experiments appear to provide the most appropriate information on slamming pressure for design use.

Recently, the overall response to slamming has been studied and thought to be of equal importance to the local effects, especially on new larger ships. In this type of study, slam pressures and forces are still of importance but

their time histories are also needed. The magnitude, duration, and shape of the slam-pulse-excitation force has eluded accurate prediction in both the experimental and theoretical fields. Most experimental efforts have been aimed at predicting pressure to aid in the design of bottom plating, while little has been done to determine force-time histories for slams.

Thus, from the considerations of the existing studies on ship overall response to bottom slamming, some general conclusions can be made. First, little attention has been given to developing a general method for evaluating, in the design stage, the response characteristics of a ship. Secondly, the possible theoretical models may become extremely complicated if all parameters are included in an exact manner, (implying that simplifications are necessary). Finally, two main technical disciplines are involved in the problem, hydrodynamics and structural mechanics, reflecting the complexity already pointed out.

Importance of Slamming to Ship Design

Information on slamming required at the initial stage of ship design is broken into two areas -- loadings and responses. The "loadings" area is a function of ship motions, sustained speed in waves, and hull form. On the other hand, the "response" area depends on the structural characteristics of the ship such as the thickness of the bottom plate forward and

bending rigidity of the entire hull [9]. These two areas are interrelated however, and cannot be treated separately in design considerations. For example, the damage of bottom plating due to slam impact is directly associated with sustained ship speed in a seaway; and hence, in determining the thickness of the bottom plate, consideration must be given to the speed expected at sea, and this in turn depends on the seakeeping characteristics of the ship.

The localized approach looks at the ship's structure where the slam occurs. Obviously, bottom plating and stiffeners must be strong enough to survive the direct impact of the load. This tends to be a fairly sophisticated hydro-aerodynamic problem with plastic structural analysis playing an important role.

The overall response to slamming includes amidship bending moments and stresses as well as vibration effects. In order to analyze these effects it is necessary to know the impact force on the hull. Slamming impact force is evaluated by integrating the distribution of the impact pressure on the ship bottom, taking into account its duration and traveling time in the longitudinal and outboard directions as the bow emerges [9]. To estimate the impact pressure, the functional relationship between the pressure and velocity must be known, (here the velocity is the vertical component of the velocity at a specific location of a ship relative to the waves).

Slamming pressure is approximately proportional to the square of the relative velocity at the instant of impact. A method for calculating slam pressures is presented in Reference [9].

With the preceding knowledge of ship stiffness and slamming in mind, the theoretical relationship between slamming and hull response as a function of stiffness will be discussed, followed by the analysis done on this relationship as the topic of this thesis.

THEORETICAL RELATIONSHIP BETWEEN SLAMMING AND HULL RESPONSE

When looking at a ship's overall response to slamming, several factors must be considered. Probably the most important factor is the bending moment due to waves. This will always be present because in order for slamming to occur, the ship must be in waves. The wave bending moment can be determined by using the conventional double integration of a static balance on a standard wave. This moment causes steady-state wave-induced bending stress in the hull.

Other factors to be kept in mind are changes in acceleration and whipping effects. The impact force delivered to the ship while slamming produces a shudder throughout the entire hull resulting in a vibratory stress called whipping stress and a sudden change in acceleration called deceleration. It has been observed through full-scale trials that only the fundamental modes of the high-frequency acceleration and whipping stresses are appreciable [9]. This is because the higher-mode vibrations die out quickly because of strong damping characteristics. The acceleration effect can cause personnel discomfort, and the whipping stresses are superimposed on the steady-state wave-induced hull stress.

The last factor to be looked at is slamming moments. This is one of the most difficult effects to determine. An expres-

sion for slamming moment can be determined, however, using normal mode theory and the property of orthogonality. The damped vertical response of a ship's hull to transient forces, assuming it behaves like a free-free nonuniform beam of length L, is governed by the following system of partial differential equations [10]:

$$\mu(x)\frac{\partial^2 y}{\partial t^2} + c(x)\frac{\partial y}{\partial t} + \frac{\partial V(x,t)}{\partial x} = P(x,t) \tag{1}$$

$$\frac{\partial M(x,t)}{\partial x} = V(x,t) + I_r(x)\frac{\partial^2 \gamma}{\partial t^2} \tag{2}$$

$$M(x,t) = EI(x)\frac{\partial \gamma(x,t)}{\partial x} \tag{3}$$

$$\frac{\partial y}{\partial x} = -\frac{V(x,t)}{KAG} + \gamma(x,t) \tag{4}$$

where

x = distance in longitudinal direction measured from origin of coordinate system

t = time variable

y = vertical elastic deflection, normal to x

μ = effective mass per unit length, or ship's mass per unit length m'(x) plus added mass per unit length m(x). In this case the added mass is taken as a function of space alone based on the ship's calm water waterline sections

c = damping coefficient per unit length

V = shear force in y-direction

P = total force per unit length due to ship-wave interaction, i.e. slam force

M = bending moment

I_r = mass moment of inertia of hull per unit length with respect to an axis normal to x-y plane

γ = component of slope of y due to bending only

EI = bending rigidity, where E = modulus of elasticity and, I = sectional area moment of inertia

KAG = shear rigidity, where K = ratio of average shear stress to shear stress at neutral axis under vertical load, A = section area and, G = shear modulus

The ship is assumed to have free ends, so that the boundary conditions are:

$$V(-L/2,t) = V(L/2,t) = M(-L/2,t) = M(L/2,t) = 0$$

An expression for slamming moment can be obtained by solving this system of equations using the following analysis from Reference [7]. Neglecting the rotary inertia term, the dynamic behavior of the beam can be treated in terms of series of responses in each of its normal modes i, which retain the important property of orthogonality with respect to the effective mass per unit length:

$$\int_{-L/2}^{L/2} \mu(x) X_i(x) X_j(x) dx = 0 \tag{5}$$

Here $X_i(x)$ is the normal mode function in arbitrary dimensionless units, and it simply represents a pattern of relative displacements along the length of the beam for a particular mode i.

A generalized coordinate with the dimensions of length $q_i(t)$ is used to define the displacement time history of the system in its i^{th} normal mode. Then the motion in a particular mode i is given by multiplying $q_i(t)$ by the dimensionless normal mode function $X_i(x)$, and the total response is finally given by summing the contributions from all the modes:

$$y(x,t) = \sum_{i=1}^{\infty} q_i(t) X_i(x) \tag{6}$$

Similarly, $M(x,t)$ and $V(x,t)$ can be represented as the product of $q_i(t)$ by a spatial weighting function $M_i(x)$ or $V_i(x)$, respectively, and the form of these functions will be determined from the analysis:

$$M(x,t) = \sum_{i=1}^{\infty} q_i(t) M_i(x) \tag{7}$$

$$V(x,t) = \sum_{i=1}^{\infty} q_i(t) V_i(x) \tag{8}$$

It can be assumed that $P(x,t)$ can be written in the following series form:

$$P(x,t) = \sum_{i=1}^{\infty} \frac{\mu(x)Q_i(t)X_i(x)}{\int_{-L/2}^{L/2} \mu(x)X_i^2(x)dx} \tag{9}$$

Multiplying both sides of equation (9) by $X_i(x)$, integrating over the ship's length, and using the orthogonality property equation (5) leads to an explicit form for the function $Q_i(t)$:

$$Q_i(t) = \int_{-L/2}^{L/2} P(x,t)X_i(x)dx \tag{10}$$

Neglecting the term involving I_r and substituting in equations (1) to (4) [where (3) and (4) may be readily combined] the series representations for y(x,t), M(x,t), V(x,t), and P(x,t), equations (6) to (9), the following three expressions are obtained:*

$$\sum_{i=1}^{\infty} \left(\mu\ddot{q}_iX_i + \dot{c}q_iX_i + q_i \frac{dV_i}{dX} - \frac{\mu Q_iX_i}{\int_{-L/2}^{L/2} \mu X_i^2 dx}\right) = 0 \tag{11}$$

---

*For simplification of notation, the functional dependencies on time and space variables are dropped, and dots are used to denote differentiations with respect to time.

$$\sum_{i=1}^{\infty} (q_i V_i - q_i \frac{dM_i}{dx}) = 0 \qquad (12)$$

$$\sum_{i=1}^{\infty} [q_i \frac{d^2 X_i}{dx^2} + q_i \frac{d}{dx} (\frac{V_i}{KAG}) - \frac{q_i M_i}{EI}] = 0 \qquad (13)$$

These equations are satisfied if each term in the summation is set equal to zero. Combining the resulting equations results in:

$$\mu \ddot{q}_i X_i + c \dot{q}_i X_i + q_i \frac{d^2}{dx^2} [EI \frac{d^2 X_i}{dx^2} + EI \frac{d}{dx} (\frac{V_i}{KAG})]$$

$$= \frac{\mu Q_i X_i}{\int_{-L/2}^{L/2} \mu X_i{}^2 dx} \qquad (14)$$

If the free motion of the beam is considered with no forcing function acting, the right-hand side of equation (14) becomes zero and, after rearranging the equation, for each normal mode:

$$\frac{\ddot{q}_i + (c/\mu)\dot{q}_i}{q_i} = - \frac{1}{\mu X_i} \frac{d^2}{dx^2} [EI \frac{d^2 X_i}{dx^2} + EI \frac{d}{dx} (\frac{V_i}{KAG})]$$

$$= - \frac{1}{\mu X_i} \frac{d^2 M_i}{dx^2} \qquad (15)$$

Since the left-hand side of equation (15) is just a function of time and the right-hand side just a function of space, it is concluded that both must be equal to a constant $-\omega_i^{\,2}$, where ω_i is the natural frequency of the i^{th} mode. This leads to:

$$\ddot{q}_i + (c/\mu)\dot{q}_i + \omega_i^{\,2} q_i = 0 \tag{16}$$

$$\frac{d^2 M_i}{dx^2} = \frac{d^2}{dx^2}\left[EI \frac{d^2 X_i}{dx^2} + EI \frac{d}{dx}\left(\frac{V_i}{KAG}\right)\right] = \mu\omega_i^{\,2} X_i \tag{17}$$

Integrating equation (17) along the length of the beam,

$$M_i = \int_{-L/2}^{x} \int_{-L/2}^{x} \mu\omega_i^{\,2} X_i \, dx \, dx \tag{18}$$

The following integral gives a more convenient representation for M_i:

$$M_i = \omega_i^{\,2} \int_{-L/2}^{x} (x-s)\mu(s) X_i(s) ds \tag{19}$$

(s denoting area)

Substituting equation (17) into (14), multiplying both sides by X_i, and taking the space integral of both sides from -L/2 to L/2, the following is obtained [using again the orthogonality principle, equation (5)]:

$$\bar{\mu}_i \ddot{q}_i + \bar{c}_i \dot{q}_i + \bar{k}_i q_i = Q_i \tag{20}$$

where $\bar{\mu}_i$, the generalized mass, is defined by

$$\bar{\mu}_i = \int_{-L/2}^{L/2} \mu X_i^2 dx \tag{21}$$

and $\bar{c}_i$, the generalized damping, is

$$\bar{c}_i = \int_{-L/2}^{L/2} c X_i^2 dx \tag{22}$$

and $\bar{k}_i$, the generalized spring constant, is

$$\bar{k}_i = \omega_i^2 \bar{\mu}_i \tag{23}$$

Since the effective mass per unit length μ is only a function of x, it follows that $\bar{\mu}_i$ is a constant for a particular mode i and it has dimensions of mass. The generalized damping $\bar{c}_i$ is, for similar reasons, also a constant. Since ω_i or the natural frequency of the i^{th} mode has a certain fixed value, it is concluded that equation (20) is a simple constant-coefficients linear second-order differential equation where the unknown is $q_i(t)$ and the forcing function is $Q_i(t)$. Assuming that at time t = 0 the beam is at rest so that $q_i(0) = \dot{q}_i(0) = 0$, the solution is then given in a closed form as:

$$q_i(t) = \int_0^t \frac{Q_i(\tau)}{\lambda_i \bar{\mu}_i} \exp\,[-(\bar{c}_i/2\bar{\mu}_i)(t-\tau)]\,\sin\,[\lambda_i(t-\tau)]d\tau \tag{24}$$

$$\text{where } \lambda_i = \sqrt{\omega_i^2 - (1/4)(\bar{c}_i/\bar{\mu}_i)^2} \tag{25}$$

Knowing $q_i(t)$ as well as the normal mode shapes and natural frequencies, it is possible to compute $M_i\ (x)$ from equation (19) and finally to obtain the bending moment from equation (7) for any location x along the ship.*

This then allows the slamming moment to be calculated if the slam forcing function is known. However, the assumption made to neglect the term involving rotary inertia I_r caused the stiffness parameters EI(x) and KAG(x) to also be neglected. The flexibility of the structure strongly influences aspects of ship hull girder response to slams, so a relationship should be found for slamming moments with hull stiffness also considered. As the main goal of this thesis, the procedure for accomplishing this task is given in the next section.

---

*Further discussion on this solution is given in Reference [7].

EXPLANATION OF THE STUDY

Computer Program

A ship hull is an extremely complex structural system, and it must be represented by a complicated mathematical model. The ship idealization used is shown in Figure 1 which was taken from Reference [6]. It consists of a double elastic axis representation of main-hull and bottom structure that reflects the bending and shear stiffness properties of the ship along its length. In addition, evenly spaced lumped masses on each axis represent both the ship mass and the added mass of water at the mass-point in question. The bottom structure elastic axis is connected to the main-hull elastic axis by rigid bulkhead links as well as by flexible bottom-structure springs representative of the transverse stiffness properties of the double bottom. This elastic axis, lumped mass idealization, rests on buoyancy springs with spring constants determined by the waterplane area of the ship at each station. Each mass point of the hull and bottom structure has one-degree-of-freedom, translation in the vertical direction. Further discussion of the Kline-Clough program can be found in References [1], [4], and [6].

Characteristics for three ships were input into the program in order to obtain slamming moment data. The ships used included; the FOTINI-L, an 800-ft. bulk carrier, the STR. EDWARD L. RYERSON, a 712-ft. Great Lakes ore carrier and the

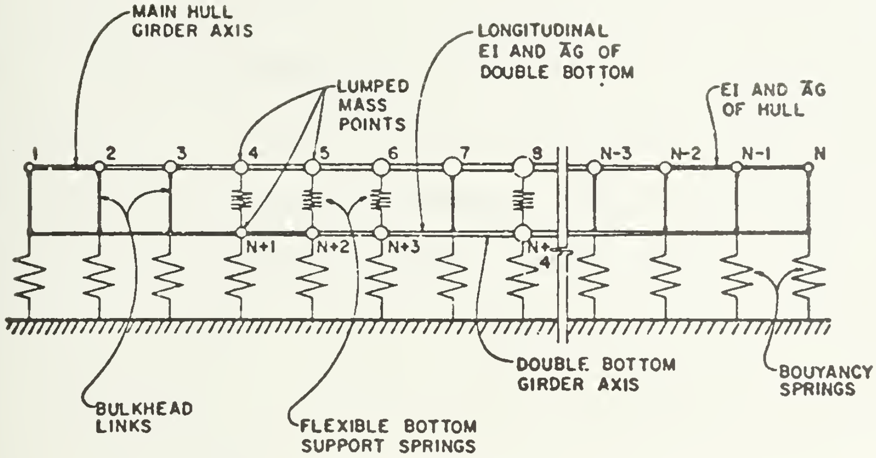


Fig. 1 Idealization of ship structure

Fig. 2 Force-time histories for slams of equal impulse

S. S. MICHIGAN, a 544-ft. general cargo ship. The principal characteristics of these ships are given in Table 1. All three ships were studied in both the loaded and ballasted condition. Hull bending stiffnesses tested were 60%, 80%, 100% (as built), 120%, and 140% of the original design. Double Bottom, Propulsion System , and Deckhouse Stiffness remained constant throughout the investigation. For each value of stiffness, it was assumed that the neutral axis remained at the same location, so the section modulus reducion was of the same percentage.

The slam force was applied to different locations on each ship. It was applied at station 2 on the FOTINI-L (36.0 ft. aft of the F.P.), station 1 on the E. L. RYERSON (17.0 ft. aft of the F.P.), and station 8 on the S. S. MICHIGAN (98.9 ft. aft of the F.P.).* In each case the force was applied to represent bow-flare type slamming, so as a result, the slam impulses were applied much nearer to the bow of each ship than is typical of bottom slams.

The slam impulse was kept constant for all tests. That is, the area under the load-time curve was constant for all three ships. From two-dimensional drop tests for a pair of forebody cross sections, it was found that, for equal vertical entry velocities, the force-time integrals for "U" and "V"

---

*The program breaks the FOTINI-L and S. S. MICHIGAN into 44 stations and the E. L. RYERSON into 42 stations.

TABLE 1 SHIP'S PRINCIPAL CHARACTERISTICS

Ship	FONTINI-L	E.L. RYERSON	S.S. MICHIGAN
Type	Bulk Carrier	Great Lakes Ore Carrier	General Cargo Ship
Displacement, tons	74,203	34,135	18,100
Overall length, ft.	820	730	579
L_{BP}, ft.	800	712	544
L_{WL}, ft.	-	730	540
Breadth, ft.	106	82	82
Depth, ft.	60.04	39.0	45.5
Design Draft, ft.-in.	44-6 1/2	26-0	27-0
Block Coefficient (L_{WL})	0.84	0.89	0.53
Section Modulus, top in.^{2} ft.	158,556	42,916	42,996*

*Section modulus for S. S. MICHIGAN found by dividing midship section moment of inertia by 55% of the depth, i.e. assuming neutral axis at 45% of the depth.

shapes did not vary greatly. As a result, it was felt that impulse was a more valid criteria of comparison from ship to ship than, say, instantaneous peak load. The impulse force-time history also had the shape of a half-sine-wave (see Figure 2).

The value of the impulse used was rather arbitrarily chosen to be 100 ton-seconds. This can be shown to be a fairly representative slam impulse, however, by calculating the resulting peak forces and slam pressures. The values of slam duration tested were 0.0625, 0.125, 0.25, 0.5, and 1.0 sec. for the FOTINI-L and E. L. RYERSON, and 0.04, 0.08, 0.16, 0.32, and 0.64 sec. for the S. S. MICHIGAN. The peak force for each slam duration can be found by integrating the half-sine-wave as follows:

it is true that,

$$\text{Impulse} = \int_0^{\tau} A \sin(\omega t)\, dt \tag{26}$$

where: A = peak force (tons)

ω = frequency of the pulse (rad/sec)

τ = slam duration (sec)

Since the pulse is a half-sine-curve, ω will always be Π divided by the slam duration. Using 0.0625 sec. as an example,

$$\omega = \frac{\Pi}{0.0625} = 50.265 \text{ radians/sec}$$

Since the value of impulse is known, substitutions can be made into equation (26) to get,

$$100 = \int_{0}^{0.0625} A \sin(50.265t)\, dt$$

Integrating gives,

$$100 = -\frac{A}{50.265} [\cos(50.265t)]_{0}^{0.0625}$$

$$= -\frac{A}{50.265} [-1-1]$$

$$\therefore \quad A = 2513.25 \text{ tons}$$

Similar calculations for the other values of slam duration yield their respective peak forces. Figure 3 is a plot of peak slam force vs. slam duration for a half-sine impulse of 100 ton-seconds.

Using a simplified approach to slam force and pressure relations, the minimum bottom area needed to produce 100 psi and 200 psi of slam pressure can be found. Comparing these chosen values of pressure to those of the MARINER operating in a Sea State 7 at a speed of 10 knots, they seem to be

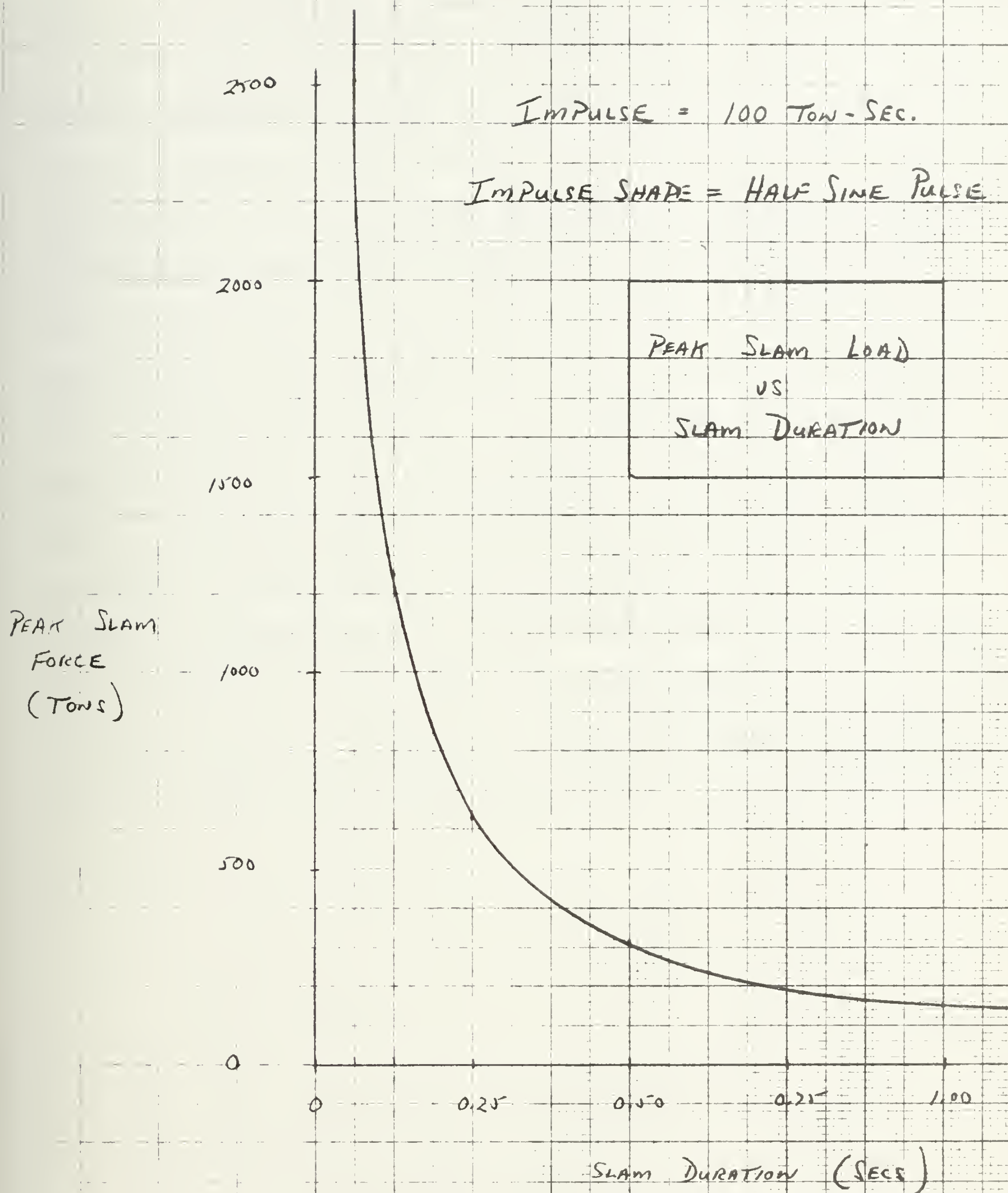

IMPULSE = 100 TON-SEC.
IMPULSE SHAPE = HALF SINE PULSE
PEAK SLAM LOAD
VS
SLAM DURATION
2500
2000
1500
1000
500
0
PEAK SLAM FORCE (TONS)
0
0.25
0.50
0.75
1.00
SLAM DURATION (SECS)

FIG. 3

reasonable. Under these conditions the most probable value of extreme slam pressure for the MARINER is 172 psi and the extreme pressure for which the probability of being exceeded is 0.10 is 218 psi [12]. Table 2 gives values of the smallest bottom area needed to attain pressures of 100 psi and 200 psi for each slam duration and coinciding peak force. For example, if the 1.0 sec. duration peak force of 157.1 tons is applied to an area larger than 12.2 ft.^{2}, the slam pressure will be less than 200 psi. At the other extreme, the peak force of 2513.25 tons (slam duration = 0.0625 sec.) must be applied to an area of 195.5 ft.^{2} or less to exceed the 200 psi "ceiling." This area could have dimensions of 15' x 13' which is reasonable for a forward bottom section which would be affected by a slam. These are very rough calculations and are meant to give a feeling for the relation between "peak slam force" and the more commonly used "extreme slam pressure."

TABLE 2*

Time Duration (secs)	Peak Force (tons)	Area Covered by Peak Force for Slam Pressures of 100 psi	200 psi
0.0625	2513.25	391.0 $ft.^2$	195.5 $ft.^2$
0.125	1256.65	195.5 $ft.^2$	97.7 $ft.^2$
0.25	628.3	97.7 $ft.^2$	48.9 $ft.^2$
0.5	314.2	48.9 $ft.^2$	24.4 $ft.^2$
1.0	157.1	24.4 $ft.^2$	12.2 $ft.^2$
0.04	3927.0	610.9 $ft.^2$	305.4 $ft.^2$
0.08	1963.5	305.4 $ft.^2$	152.7 $ft.^2$
0.16	981.8	152.7 $ft.^2$	76.4 $ft.^2$
0.32	490.9	76.4 $ft.^2$	38.2 $ft.^2$
0.64	245.4	38.2 $ft.^2$	19.1 $ft.^2$

*For half-sine impulse of 100 ton-seconds.

Procedure and Results

Before the investigation was started, several assumptions were made. First, simple beam theory is used so all assumptions associated with it are applicable. Second, it is assumed that all midship section scantlings included in moment of inertia calculations will extend at least 40% of the ship's length, centered approximately amidships. Third, whipping stresses are neglected for this study. Fourth, the effect of shear rigidity is neglected. This is a good assumption for an overall slam response investigation since the effect is not significant for the first two modes of vibration of a slender ship [9]. Finally, since ship hull flexibility is determined primarily by the demands of longitudinal strength and since longitudinal strength is determined on the basis of wave bending moment, the bending moment was judged to be the most meaningful response parameter for study. More explicitly, the amidships' bending moment was chosen as the factor to be studied.

There were several reasons for choosing the FOTINI-L, E. L. RYERSON, and S. S. MICHIGAN for this analysis. As well as having significantly different characteristics (see Table 1), these ship types are prevalent in U.S. shipping today. Also current trends in the design of bulk carriers, Great Lakes ore carriers and general cargo ships are likely to alter their

hull stiffnesses. The last and most important reason is that the appropriate data were readily available for these ships.

The computer program output graphs of slamming moment versus station, at time of maximum moment amidships for the different values of slam duration and hull stiffness. Examples of these plots are shown in the Appendix. The graphs were used to find the maximum bending moment amidships due to slamming. On some of the plots, the highest moment was one or two stations from amidships. In this case, the larger moment was read. A summary of these slamming moments for each ship can be found in Tables 3, 4, and 5. The moments were then plotted against slam duration for each value of hull stiffness, see Figures 4a, 4b, 5a, 5b, 6a, and 6b (moments were also plotted against peak force for the FOTINI-L in Figure 4c and 4d but not for the other ships since it was felt that slam duration was a more appropriate parameter for this study). Note that graphs are made for each ship in both the loaded and ballasted condition.

The section modulus of each ship was calculated for the changes in stiffness, assuming that the moment of inertia changed by the same percentage. Table 6 lists each ship's section modulus (to the deck) as it changes with stiffness. Deck stresses were then calculated using simple beam theory. Tables 7, 8, and 9 list the resulting stresses. Graphs of deck stress due to slamming versus slam duration are shown

TABLE 3

Maximum Slam Moments Amidships - tons feet ($x10^{-3}$)

FOTINI-L

Full Load Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	70.5	67.5	57.3	39.2	24.6
80	75.0	72.0	61.2	41.6	25.8
100	-	74.4	63.6	-	26.4
120	82.8	77.4	65.7	44.4	27.0
140	84.9	80.4	67.2	45.4	27.1

Ballast Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	62.4	60.0	52.0	38.0	24.2
80	68.7	65.7	56.8	40.4	25.2
100	72.6	69.6	60.0	42.0	26.1
120	77.1	72.9	63.0	43.8	26.4
140	80.1	76.8	65.4	44.8	26.5

TABLE 4

Maximum Slam Moments Amidships - tons feet ($x10^{-3}$)

E. L. RYERSON

Full Load Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	78.1	67.0	48.8	36.9	26.9
80	87.5	72.5	54.8	42.0	27.2
100	95.3	77.8	58.1	45.0	28.0
120	96.2	82.5	58.6	45.6	27.8
140	103.1	83.8	62.8	46.5	28.1

Ballast Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	85.0	70.0	52.5	42.9	26.9
80	97.8	77.0	54.0	48.8	27.5
100	103.4	82.5	58.1	50.3	28.4
120	109.4	85.3	60.0	51.5	29.1
140	116.3	90.6	66.3	-	30.0

TABLE 5

Maximum Slam Moments Amidships - tons feet ($\times 10^{-3}$)

S. S. MICHIGAN

Full Load Condition

Slam Duration (sec)

EI	0.04	0.08	0.16	0.32	0.64
60	45.3	39.4	31.0	22.5	15.1
80	50.9	44.3	33.8	24.1	15.2
100	56.2	47.5	36.3	25.3	15.4
120	58.3	51.0	38.1	26.3	15.7
140	62.2	53.8	39.6	26.9	15.8

Ballast Condition

Slam Duration (sec)

EI	0.04	0.08	0.16	0.32	0.64
60	62.6	58.1	44.8	30.0	17.6
80	71.3	63.8	48.3	31.8	17.7
100	78.8	69.4	51.0	32.8	18.0
120	87.8	71.4	53.0	33.8	18.0
140	89.5	76.5	57.3	34.5	17.3

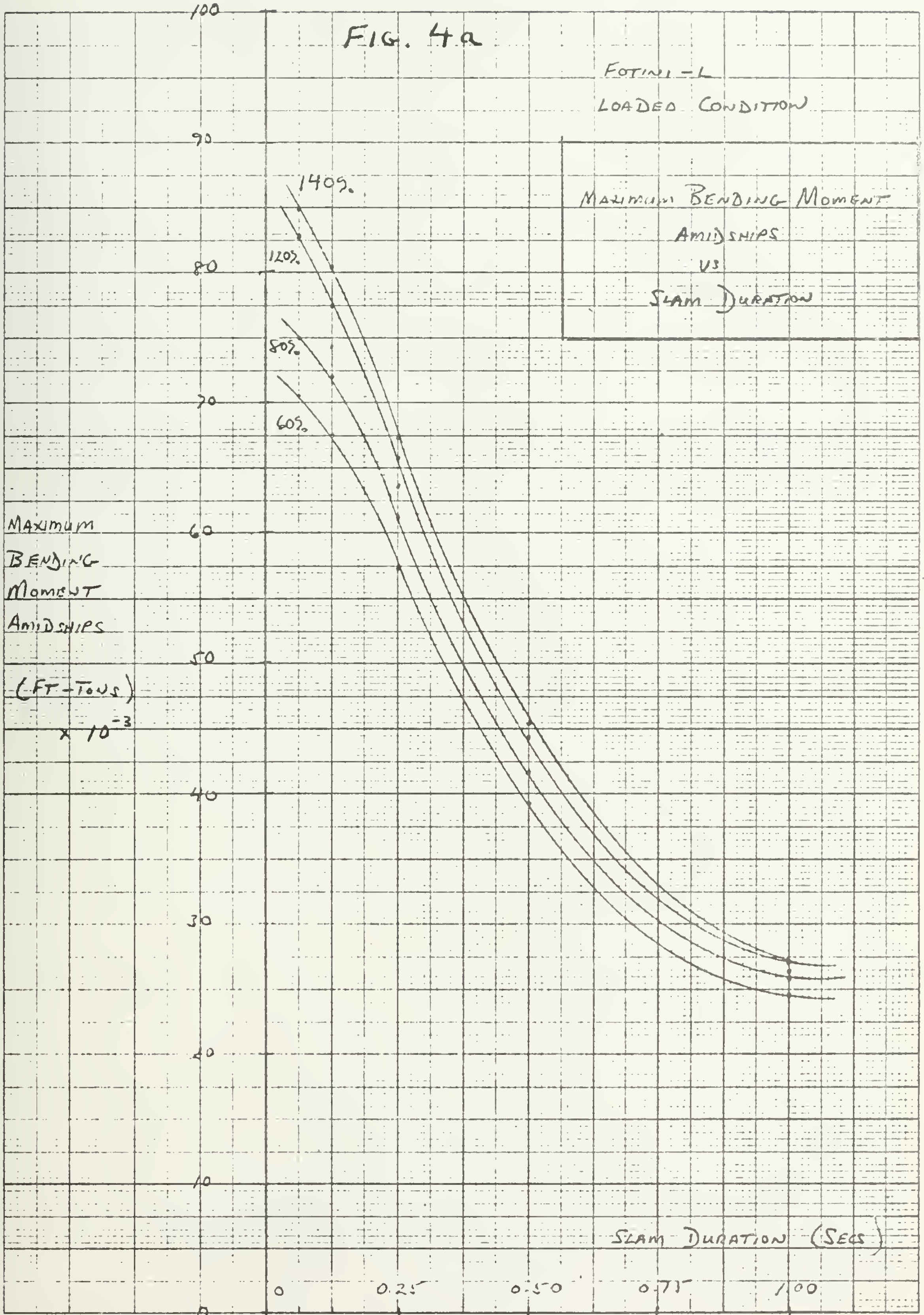


FIG. 4a
FOTINI-L
LOADED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SLAM DURATION
140%
120%
80%
60%
MAXIMUM
BENDING
MOMENT
AMIDSHIPS
(FT-TONS)
x 10^-3
100
90
80
70
60
50
40
30
20
10
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

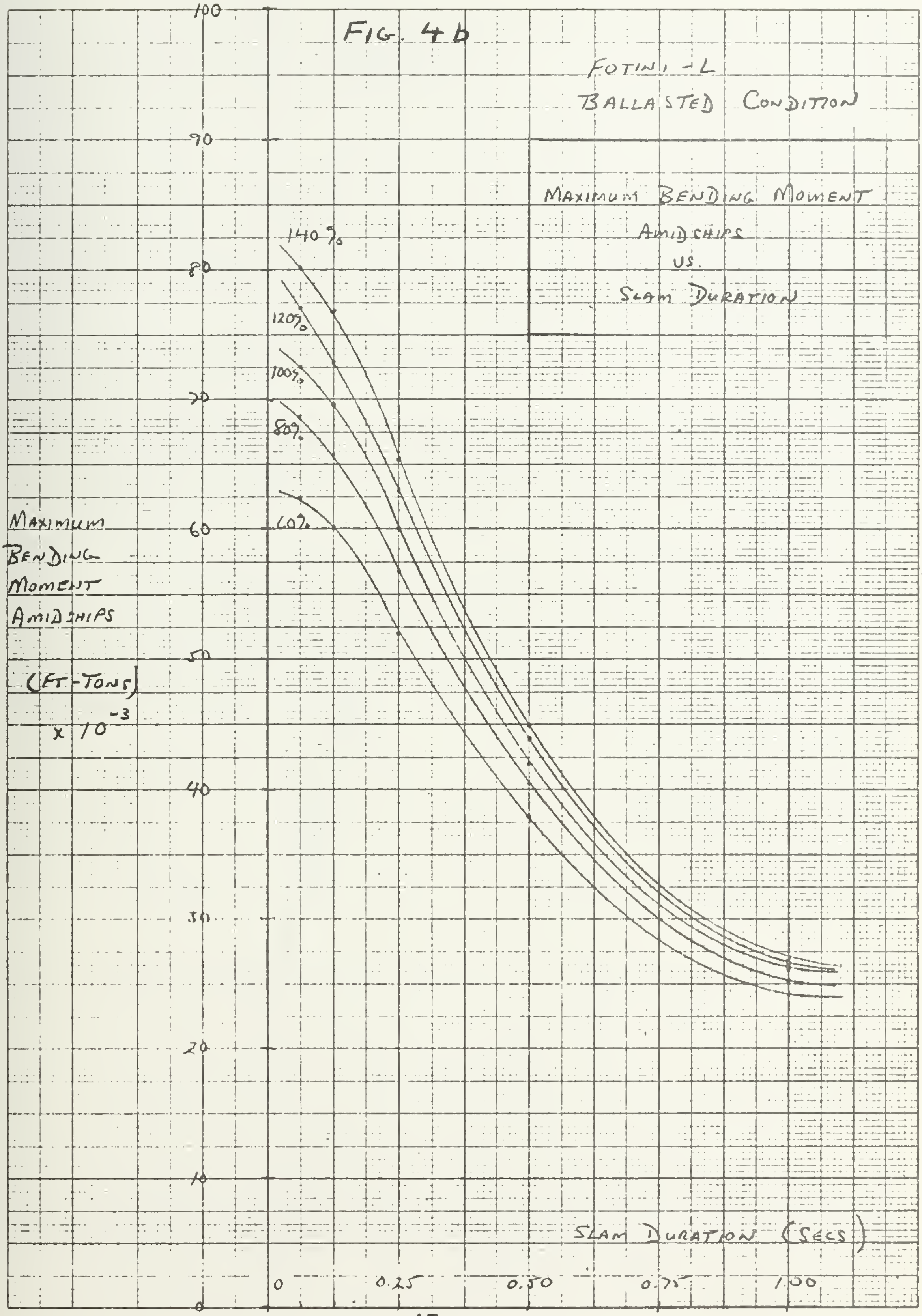


FIG. 4b
FOTINI - L
BALLASTED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS.
SLAM DURATION
140%
120%
100%
80%
60%
MAXIMUM
BENDING
MOMENT
AMIDSHIPS
(FT-TONS)
x 10^-3
100
90
80
70
60
50
40
30
20
10
0
0
0.25
0.50
0.75
1.00
SLAM DURATION (SECS)

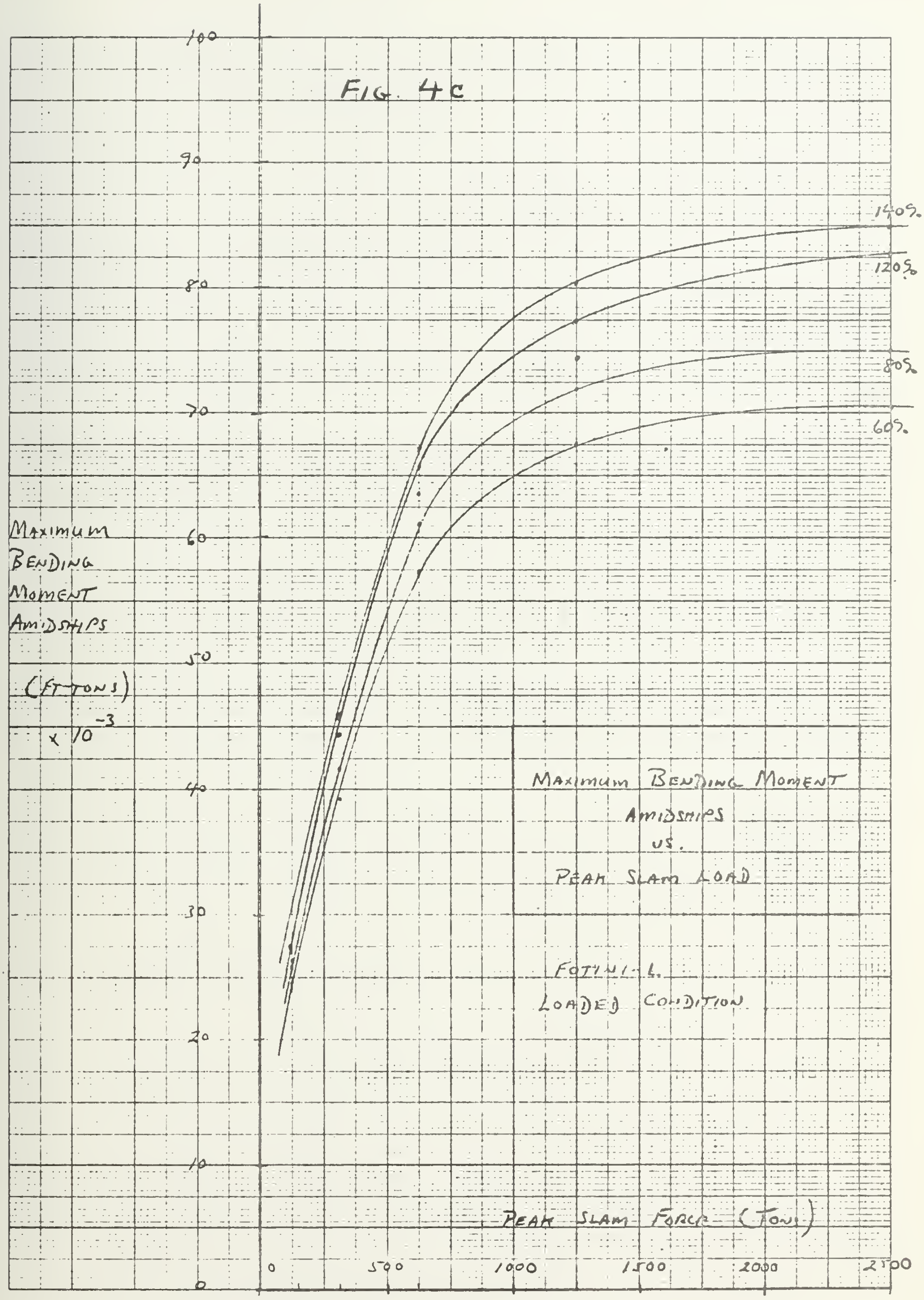


FIG. 4c
Maximum Bending Moment Amidships (FT-TONS) x 10^-3
100
90
80
70
60
50
40
30
20
10
0
140%
120%
80%
60%
Maximum Bending Moment Amidships vs. Peak Slam Load
Fotini-L.
Loaded Condition
Peak Slam Force (Tons)
0
500
1000
1500
2000
2500

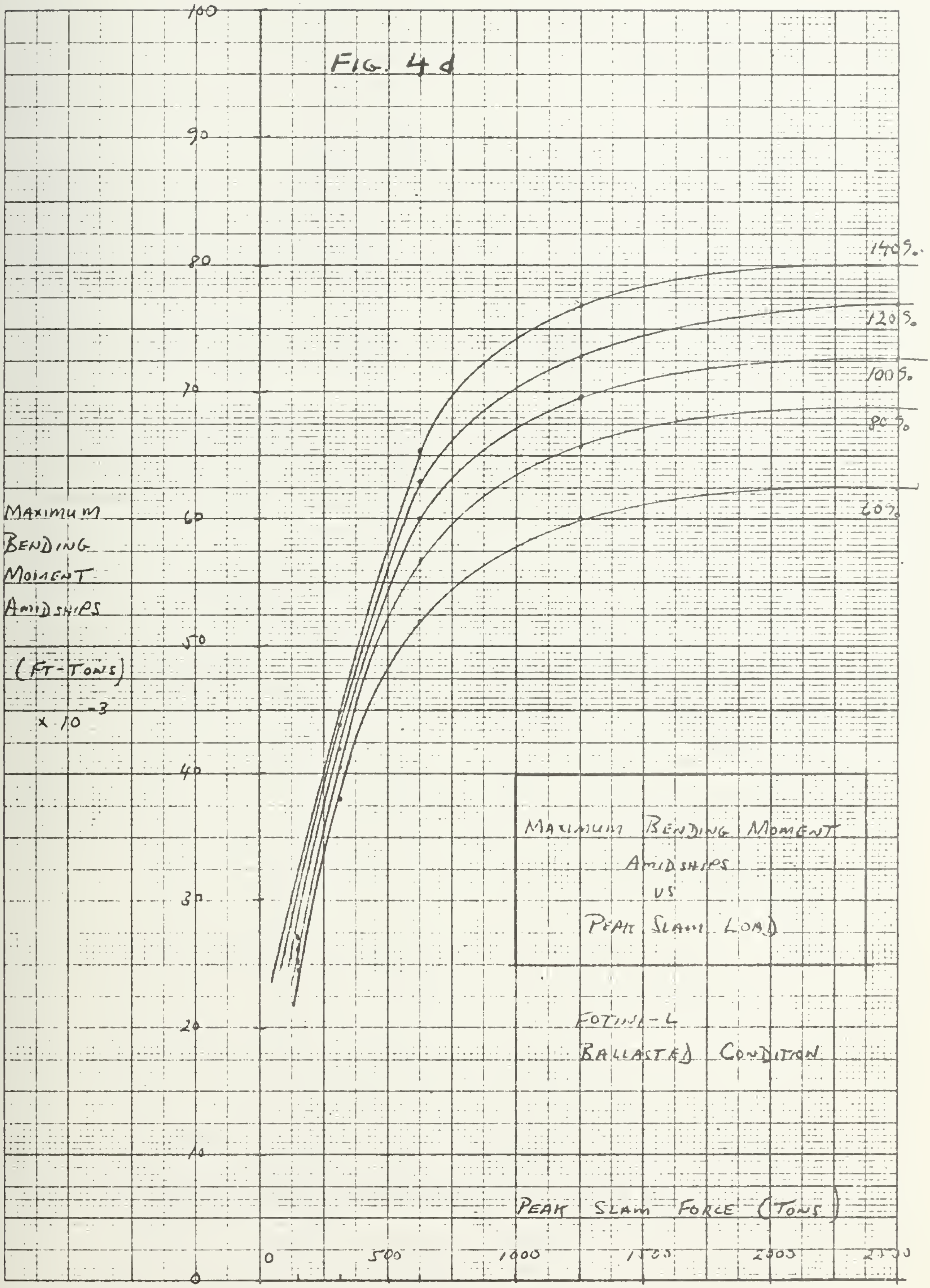


FIG. 4d
MAXIMUM BENDING MOMENT AMIDSHIPS (FT-TONS) x 10^-3
PEAK SLAM FORCE (TONS)
MAXIMUM BENDING MOMENT AMIDSHIPS VS PEAK SLAM LOAD
FOTINI-L
BALLASTED CONDITION
140%
120%
100%
80%
60%
100
90
80
70
60
50
40
30
20
10
0
0
500
1000
1500
2000

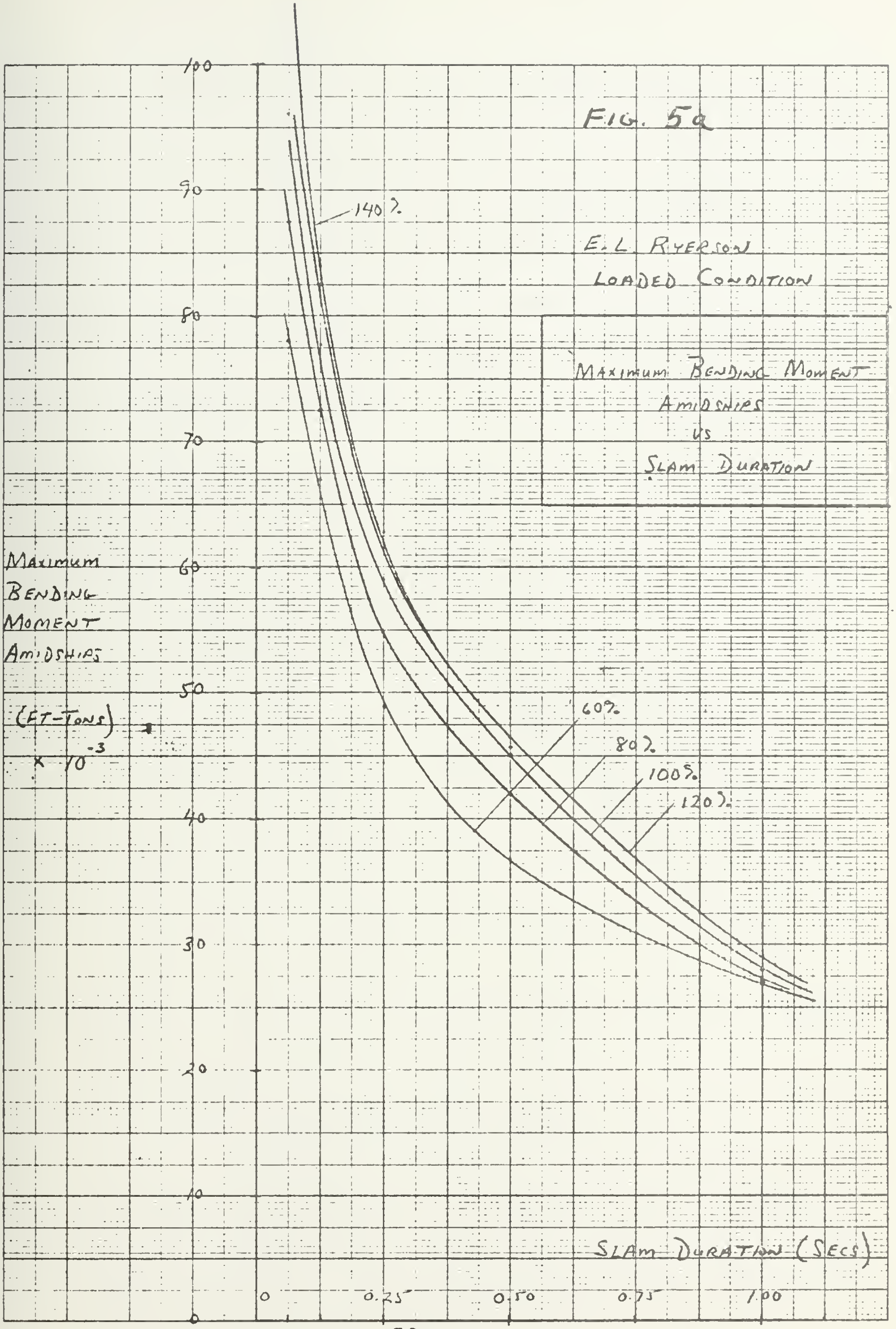


FIG. 5a
E. L. RYERSON
LOADED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SLAM DURATION
MAXIMUM BENDING MOMENT AMIDSHIPS (FT-TONS) × 10-3
SLAM DURATION (SECS)
140%
60%
80%
100%
120%
100
90
80
70
60
50
40
30
20
10
0
0
0.25
0.50
0.75
1.00

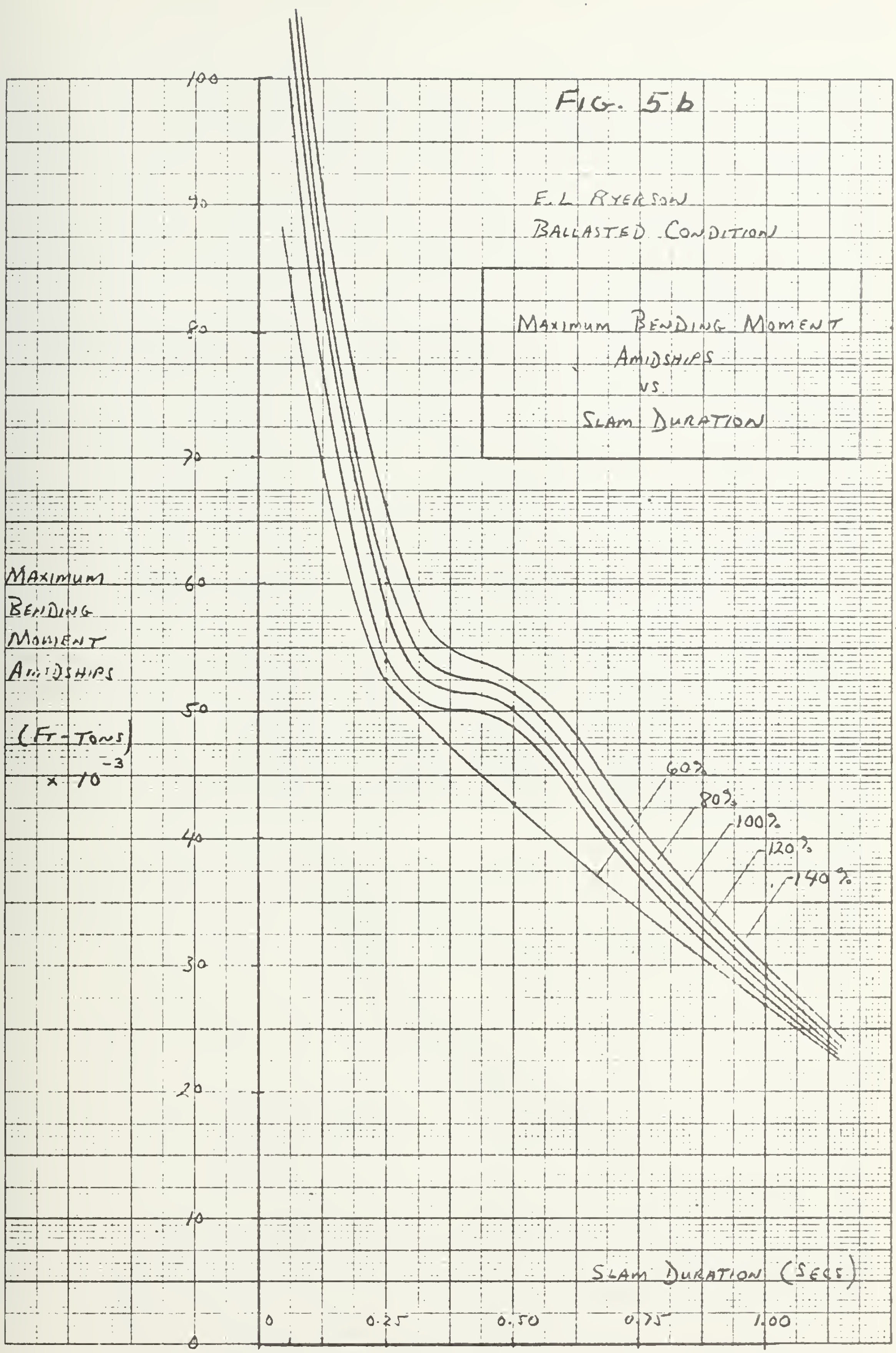


FIG. 5b
E.L. RYERSON
BALLASTED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SLAM DURATION
MAXIMUM
BENDING
MOMENT
AMIDSHIPS
(FT-TONS)
x 10^-3
100
90
80
70
60
50
40
30
20
10
0
60%
80%
100%
120%
140%
0
0.25
0.50
0.75
1.00
SLAM DURATION (SECS)

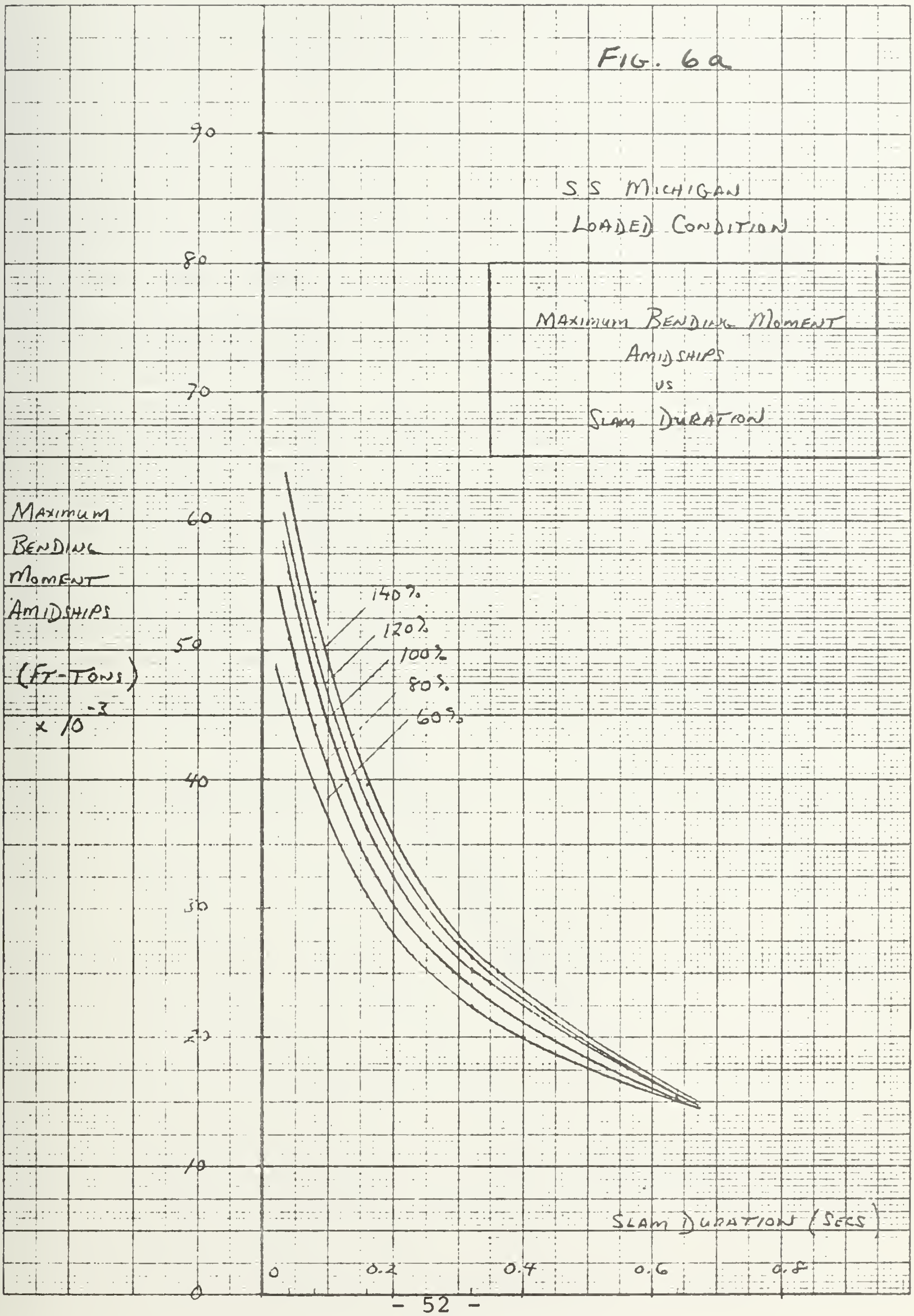


FIG. 6a
S S MICHIGAN
LOADED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SLAM DURATION
MAXIMUM
BENDING
MOMENT
AMIDSHIPS
(FT-TONS)
x 10^-3
140%
120%
100%
80%
60%
90
80
70
60
50
40
30
10
0
0
0.2
0.4
0.6
0.8
SLAM DURATION (SECS)

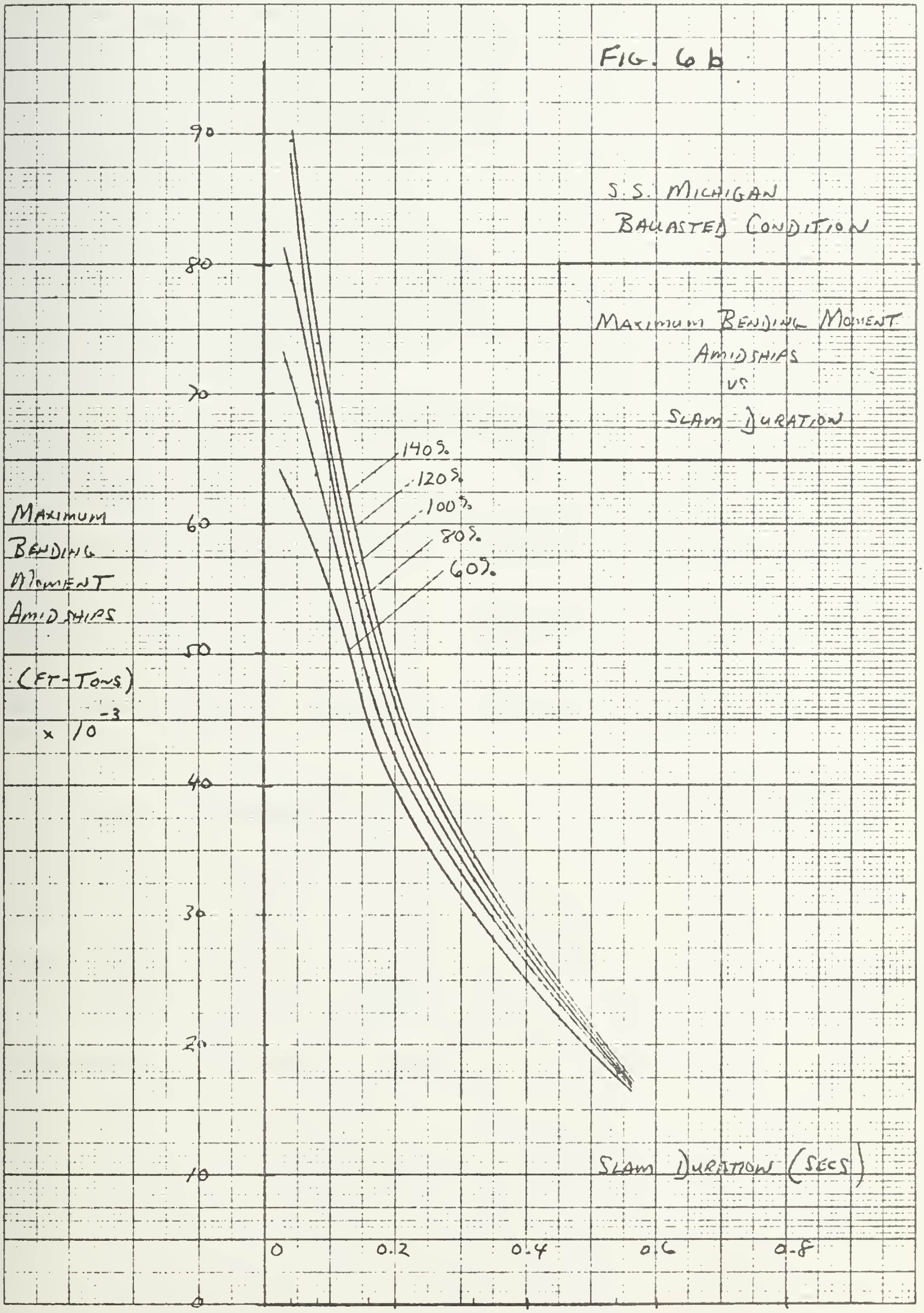


FIG. 6b
S.S. MICHIGAN
BALLASTED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SLAM DURATION
MAXIMUM
BENDING
MOMENT
AMIDSHIPS
(FT-TONS)
x 10^-3
140%
120%
100%
80%
60%
90
80
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
SLAM DURATION (SECS)

in Figures 7a, 7b, 8a, 8b, 9a, and 9b (again, stress was plotted against peak force for the FOTINI-L in Figures 7c and 7d but not for the other ships). These and the bending moment graphs will be discussed in the "Conclusions" section of the thesis.

TABLE 6

Section Modulus Amidships (to the deck) - in.^{2}ft.

Stiffness, EI

SHIP	60	80	100	120	140
FOTINI-L	95,133.6	126,844.8	158,556.0	190,267.2	221,978.4
E. L. RYERSON	25,749.6	34,332.8	42,916.0	51,499.2	60,082.4
S. S. MICHIGAN	25,797.6	34,396.8	42,996.0	51,595.2	60,194.4

Since a relationship between stiffness and slam moment was desired, the moments were plotted against hull stiffness for each slam duration. These graphs are illustrated in Figures 10a, 10b, 11a, 11b, 12a, and 12b. From these plots it was concluded that the slamming moment could be related to hull stiffness by the general equation,

$$\text{Slam Moment} = A\,(EI)^b + C \tag{27}$$

TABLE 7

Maximum Slam Stress Amidships - tons/in.^{2}

FOTINI-L

Full Load Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	0.741	0.710	0.602	0.412	0.259
80	0.591	0.568	0.482	0.328	0.203
100	-	0.469	0.401	-	0.167
120	0.435	0.407	0.345	0.233	0.142
140	0.382	0.362	0.303	0.205	0.122

Ballast Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	0.656	0.631	0.547	0.400	0.254
80	0.542	0.518	0.448	0.318	0.199
100	0.458	0.439	0.378	0.265	0.165
120	0.405	0.383	0.331	0.230	0.139
140	0.361	0.346	0.295	0.202	0.119

TABLE 8

Maximum Slam Stress Amidships - tons/in.^{2}

E. L. RYERSON

Full Load Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	3.03	2.60	1.90	1.43	1.04
80	2.55	2.11	1.60	1.22	0.79
100	2.22	1.81	1.35	1.05	0.65
120	1.87	1.60	1.14	0.89	0.54
140	1.72	1.39	1.05	0.77	0.47

Ballast Condition

Slam Duration (sec)

EI	0.0625	0.125	0.25	0.5	1.0
60	3.30	2.72	2.04	1.67	1.04
80	2.85	2.24	1.57	1.42	0.80
100	2.41	1.92	1.35	1.17	0.66
120	2.12	1.66	1.17	1.00	0.56
140	1.94	1.51	1.10	-	0.50

TABLE 9

Maximum Slam Stress Amidships - tons/in.^{2}

S. S. MICHIGAN

Full Load Condition

Slam Duration (sec)

EI	0.04	0.08	0.16	0.32	0.64
60	1.76	1.53	1.20	0.87	0.59
80	1.48	1.29	0.98	0.70	0.44
100	1.31	1.10	0.84	0.59	0.36
120	1.13	0.99	0.74	0.51	0.30
140	1.03	0.89	0.66	0.45	0.26

Ballast Condition

Slam Duration (sec)

EI	0.04	0.08	0.16	0.32	0.64
60	2.43	2.25	1.74	1.16	0.68
80	2.07	1.85	1.40	0.92	0.51
100	1.83	1.61	1.19	0.76	0.42
120	1.70	1.38	1.03	0.66	0.35
140	1.49	1.27	0.95	0.57	0.29

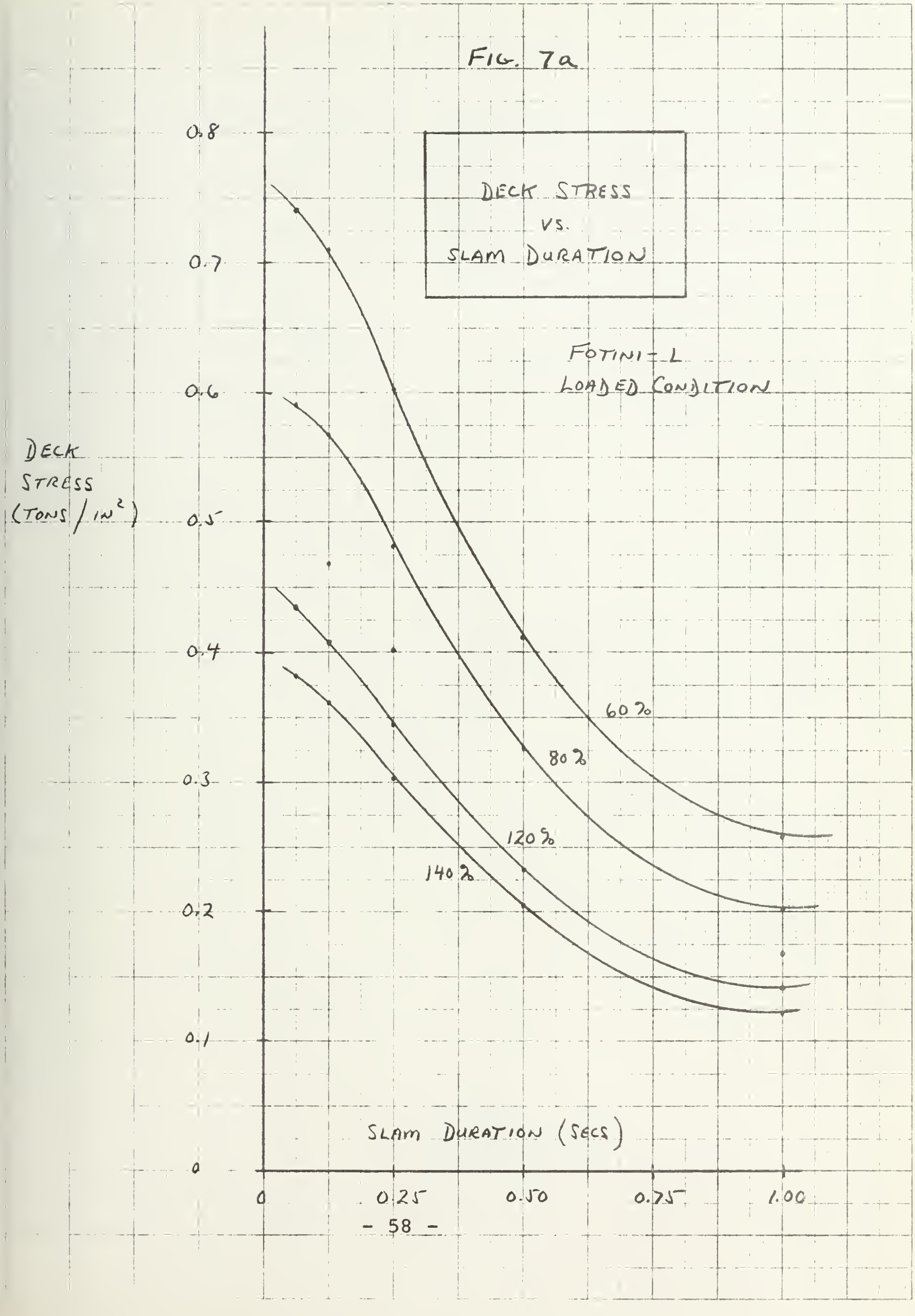
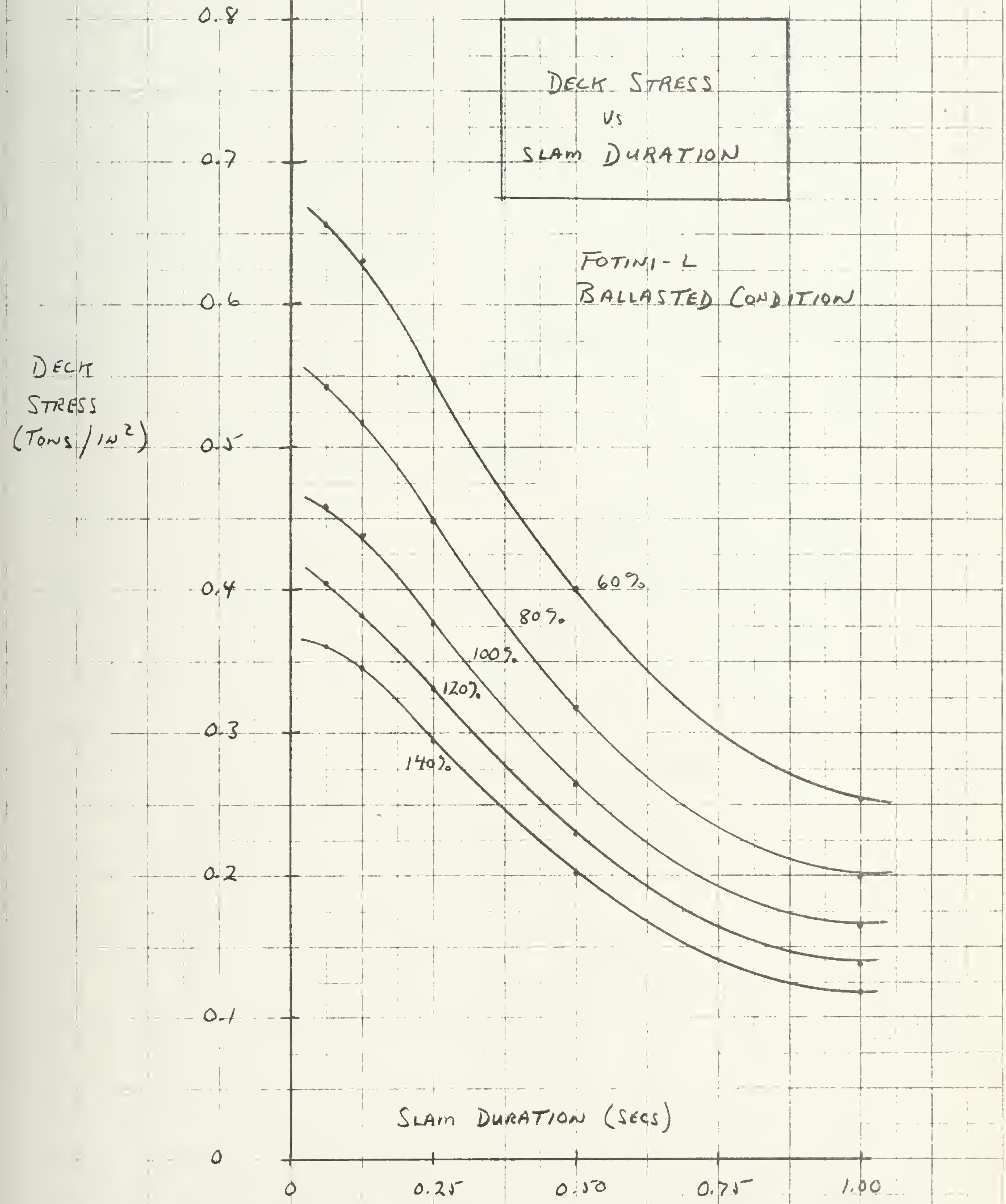


FIG. 7a
DECK STRESS
VS.
SLAM DURATION
FOTINI-L
LOADED CONDITION
DECK
STRESS
(TONS / IN²)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
60%
80%
120%
140%
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

FIG. 7b


DECK STRESS
VS
SLAM DURATION
FOTINI - L
BALLASTED CONDITION
DECK STRESS (TONS / IN²)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
60%
80%
100%
120%
140%
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

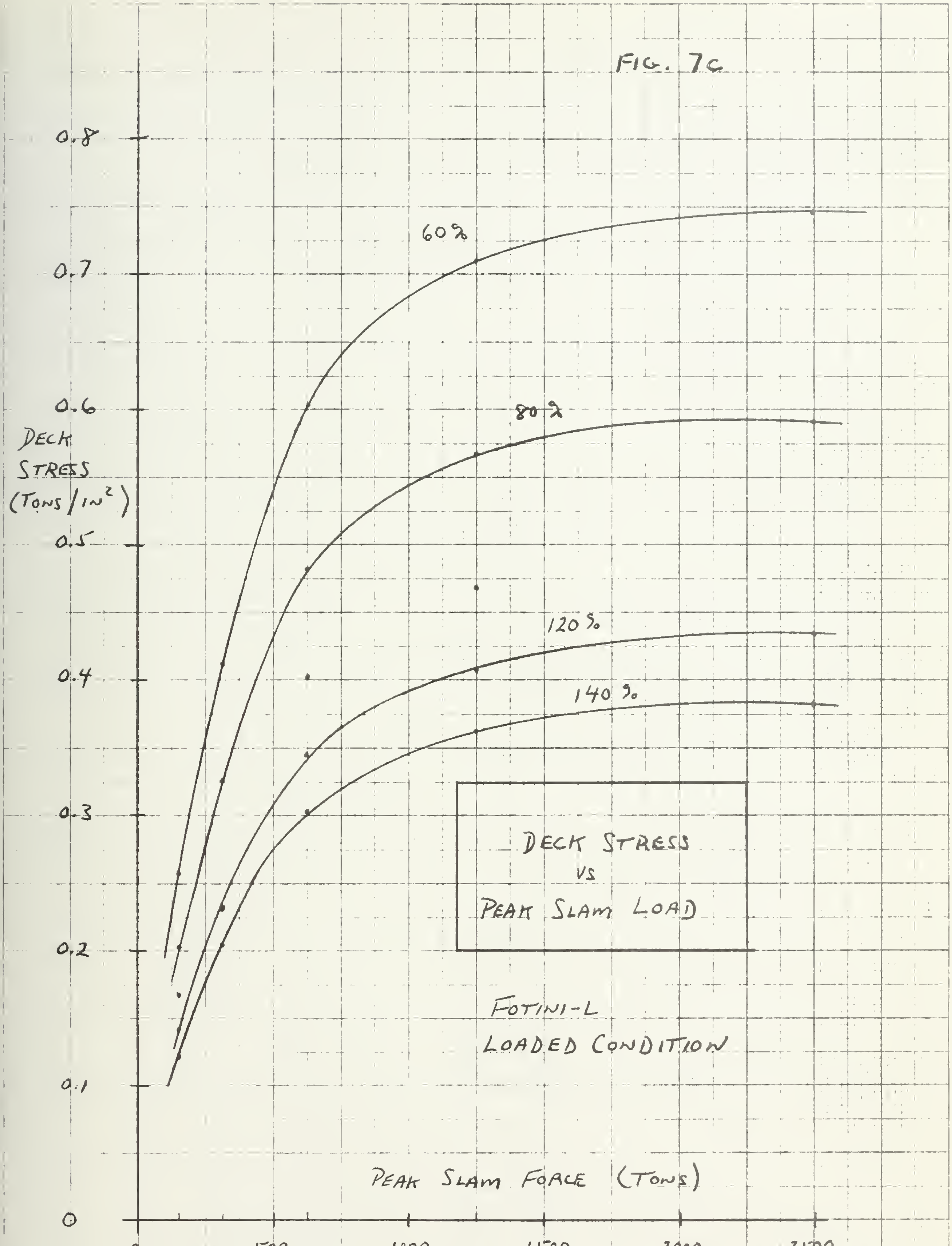


FIG. 7c
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
DECK STRESS (TONS / IN²)
60%
80%
120%
140%
DECK STRESS VS PEAK SLAM LOAD
FOTINI-L
LOADED CONDITION
PEAK SLAM FORCE (TONS)
0
500
1000
1500
2000
2500

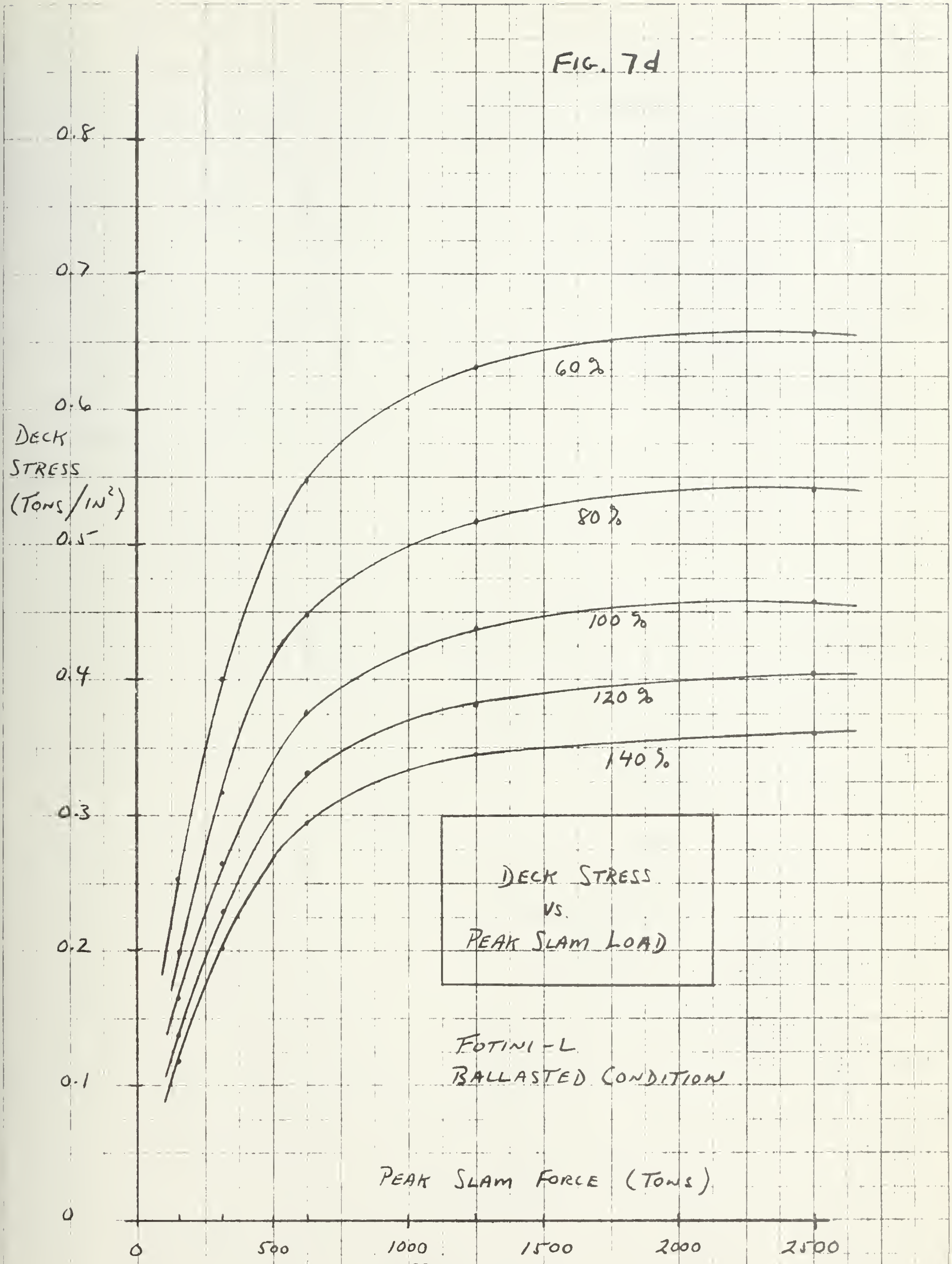


FIG. 7d
DECK STRESS (TONS/IN²)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
60%
80%
100%
120%
140%
DECK STRESS VS. PEAK SLAM LOAD
FOTINI-L
BALLASTED CONDITION
PEAK SLAM FORCE (TONS)
0
500
1000
1500
2000
2500

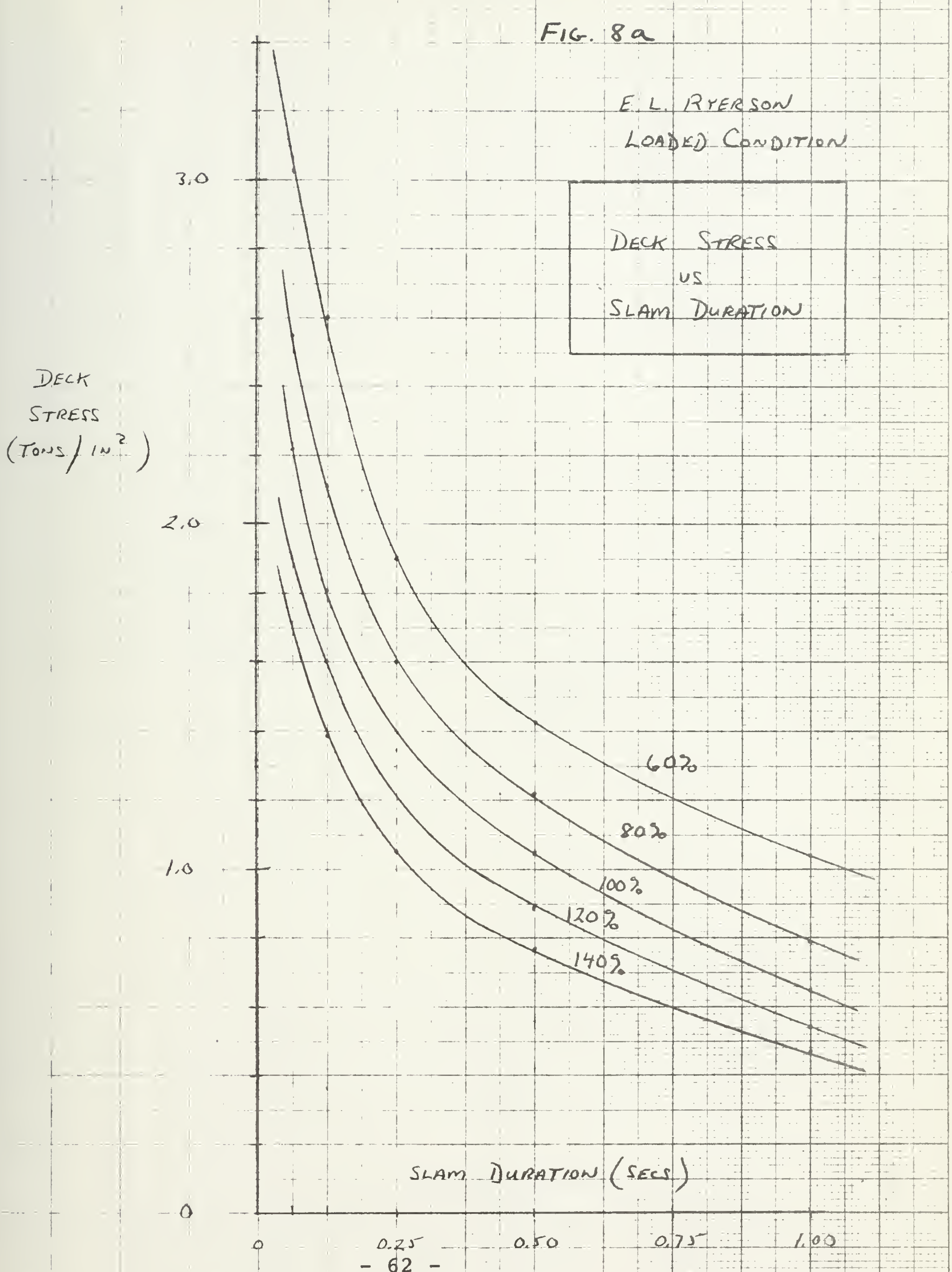


FIG. 8a
E. L. RYERSON
LOADED CONDITION
DECK STRESS
VS
SLAM DURATION
DECK STRESS (TONS / IN²)
3.0
2.0
1.0
0
60%
80%
100%
120%
140%
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

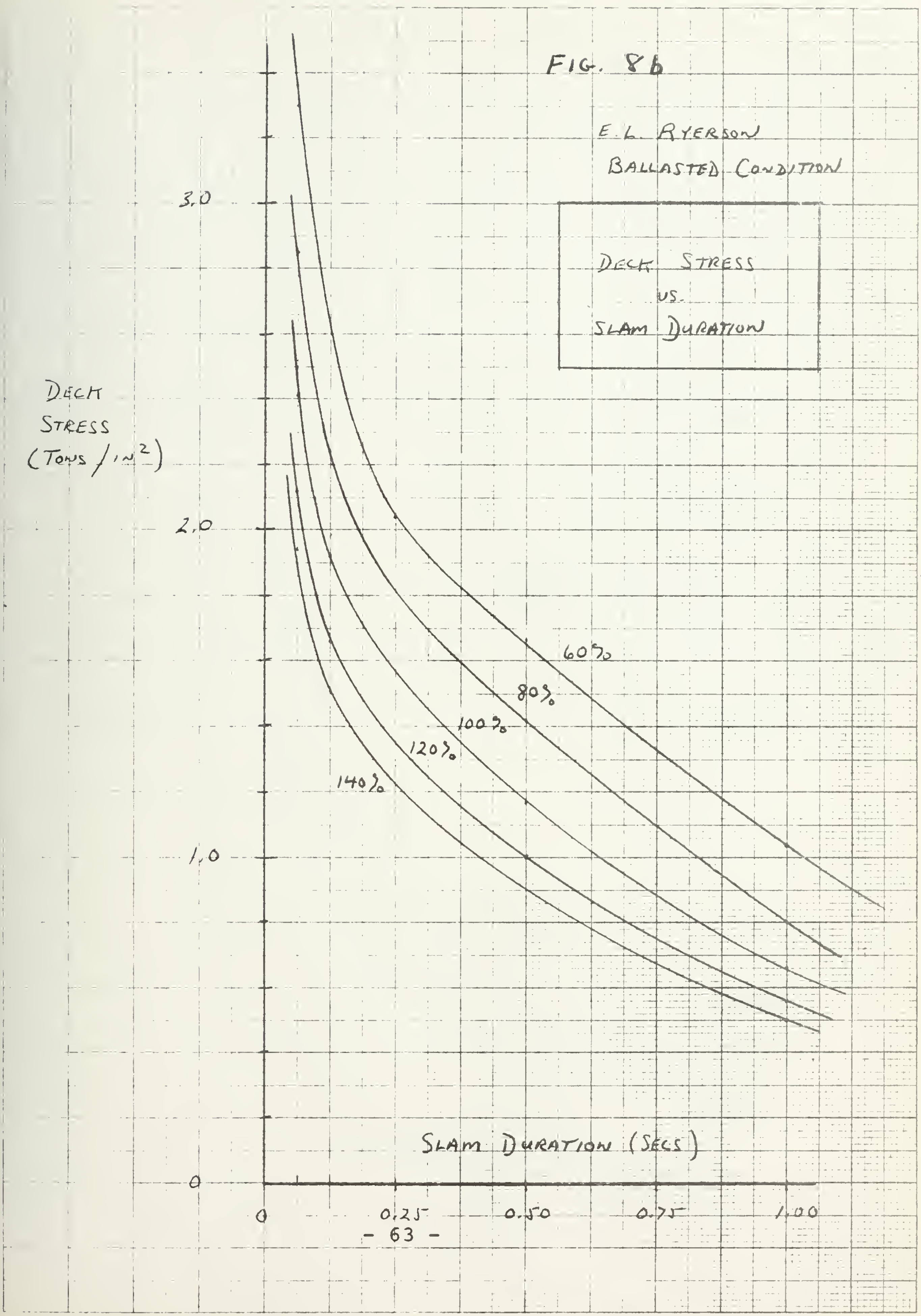


FIG. 8b
E.L. RYERSON
BALLASTED CONDITION
DECK STRESS
VS.
SLAM DURATION
DECK STRESS (TONS / IN²)
3.0
2.0
1.0
0
60%
80%
100%
120%
140%
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

FIG. 9a

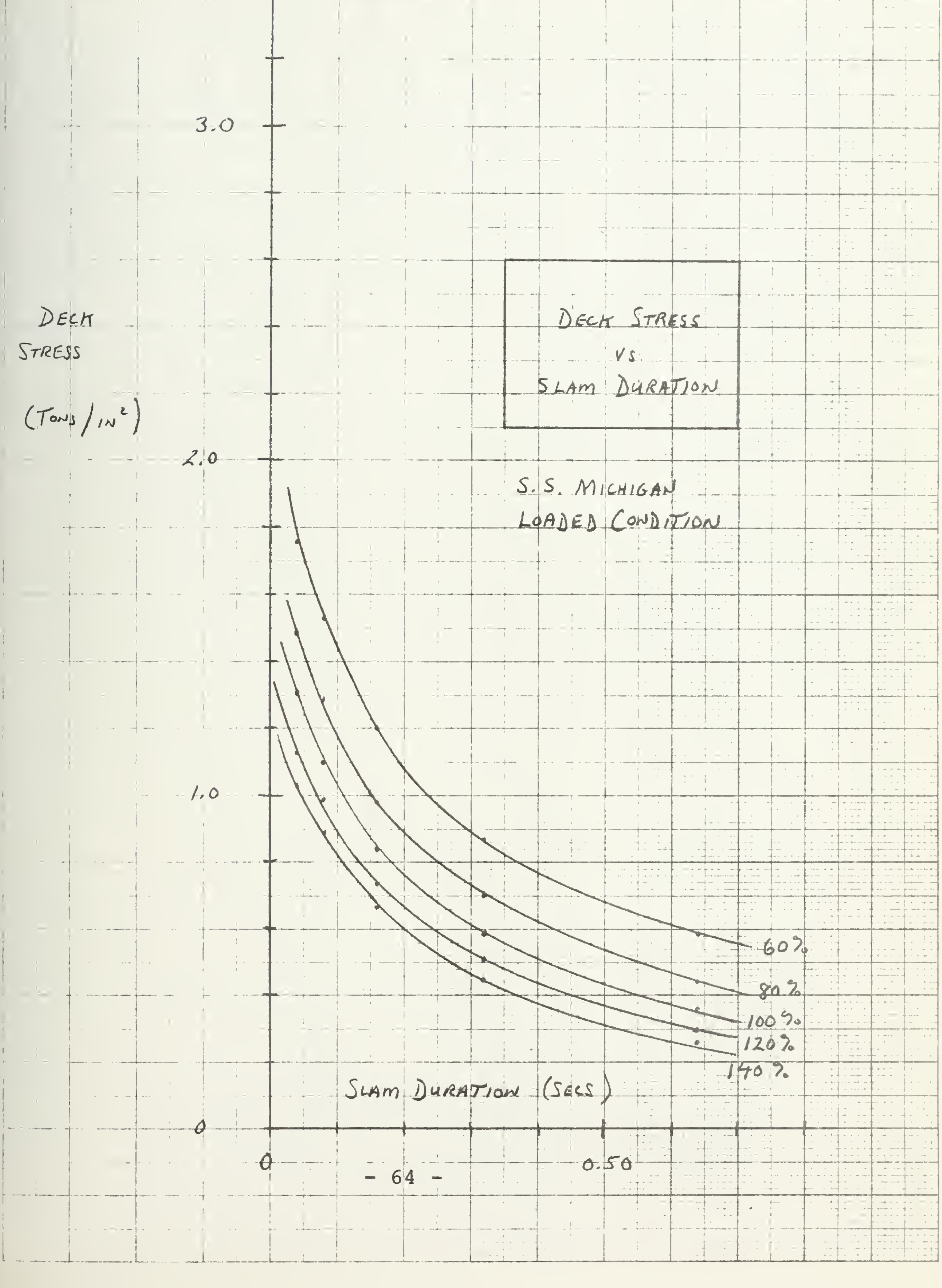
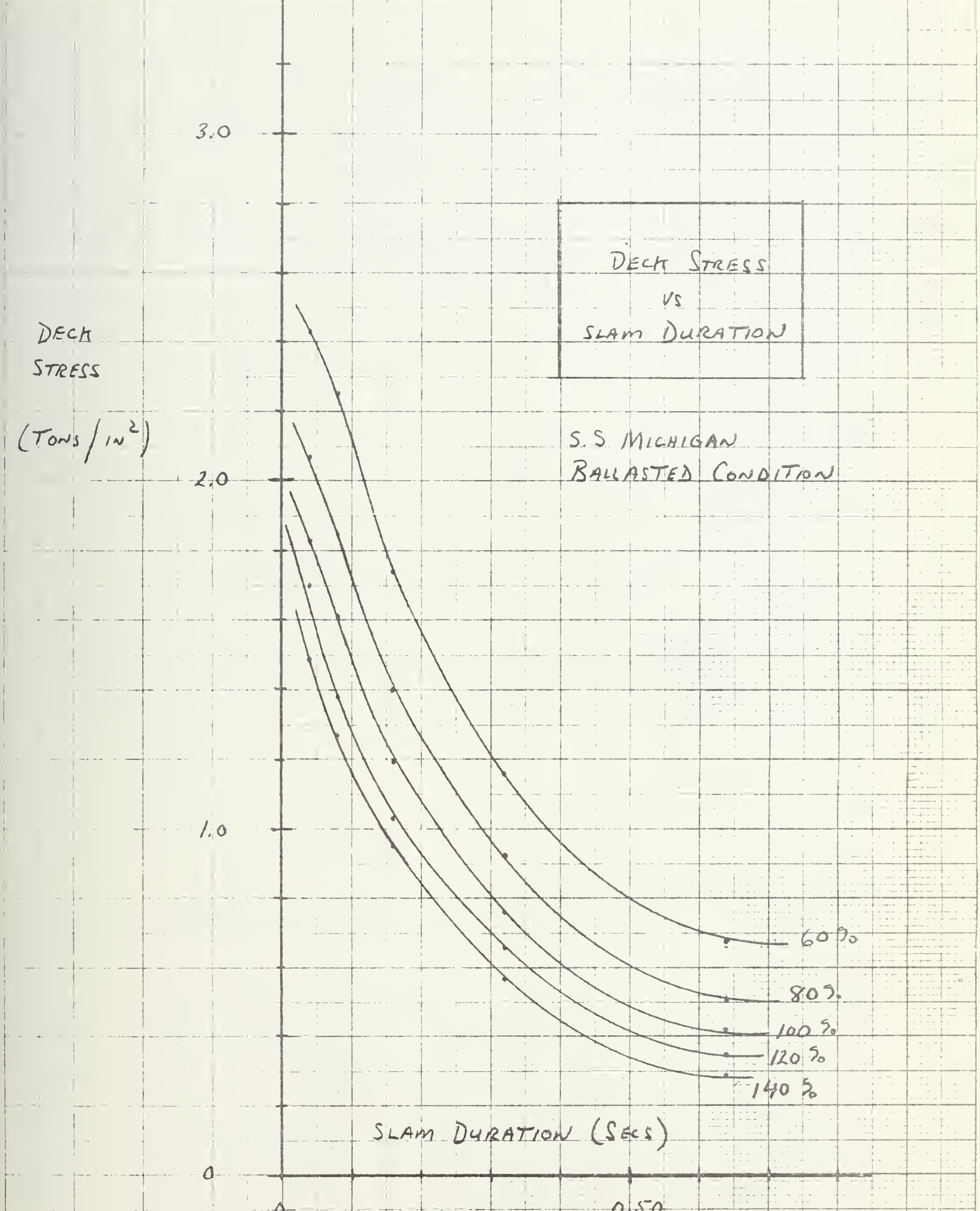
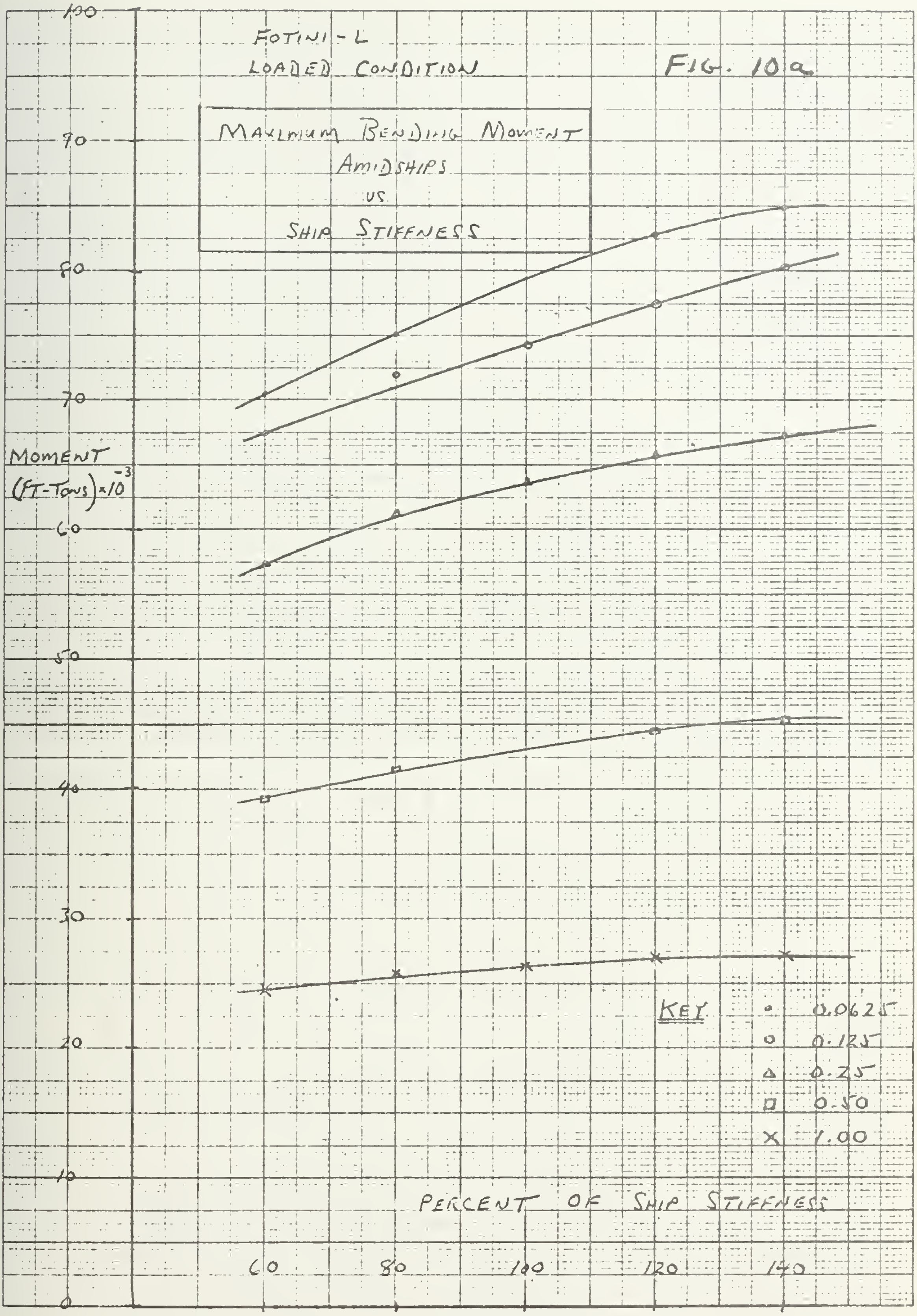
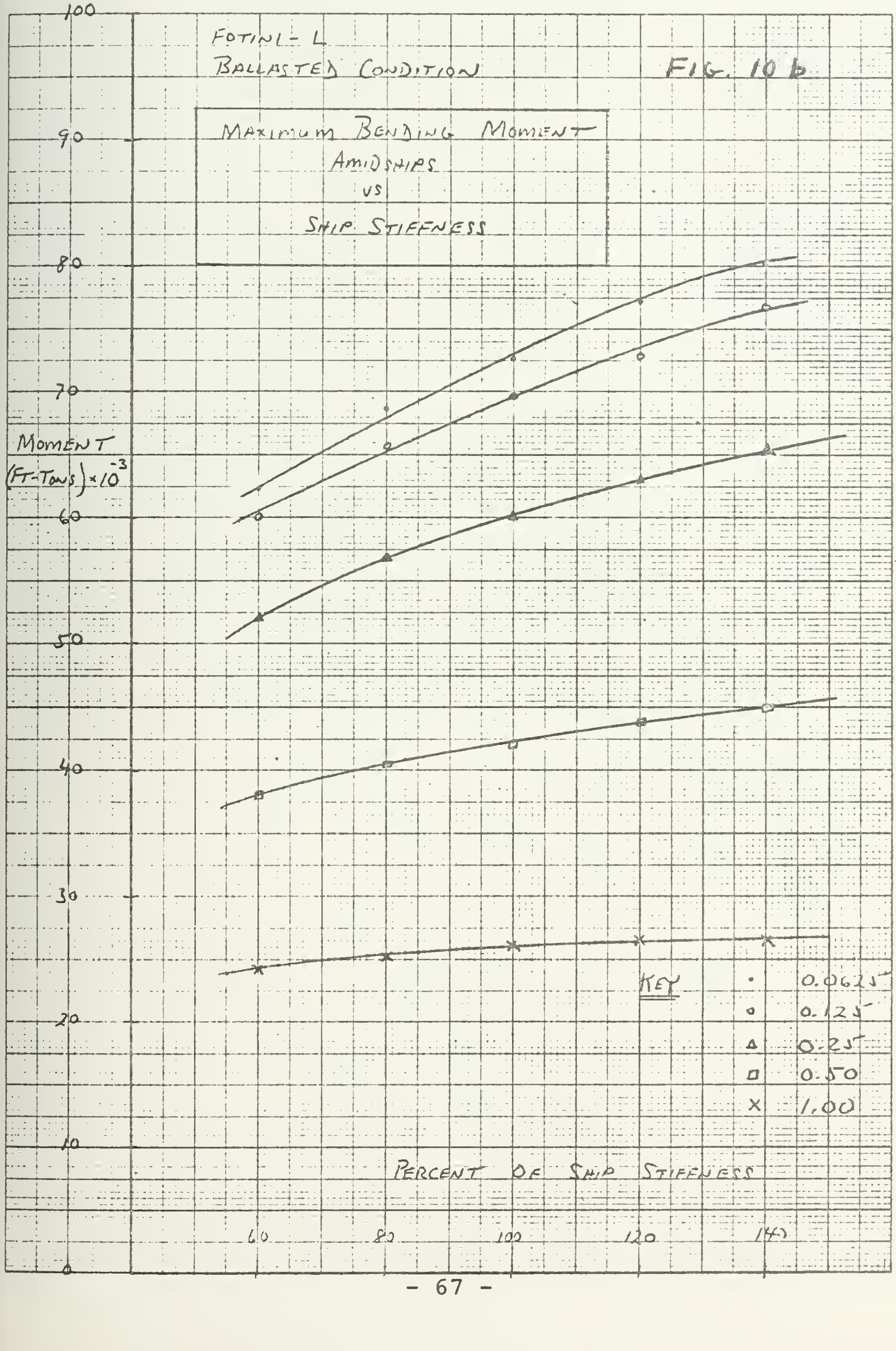

DECK STRESS VS SLAM DURATION
S.S. MICHIGAN
LOADED CONDITION
DECK STRESS (TONS/IN²)
3.0
2.0
1.0
0
60%
80%
100%
120%
140%
SLAM DURATION (SECS)
0
0.50

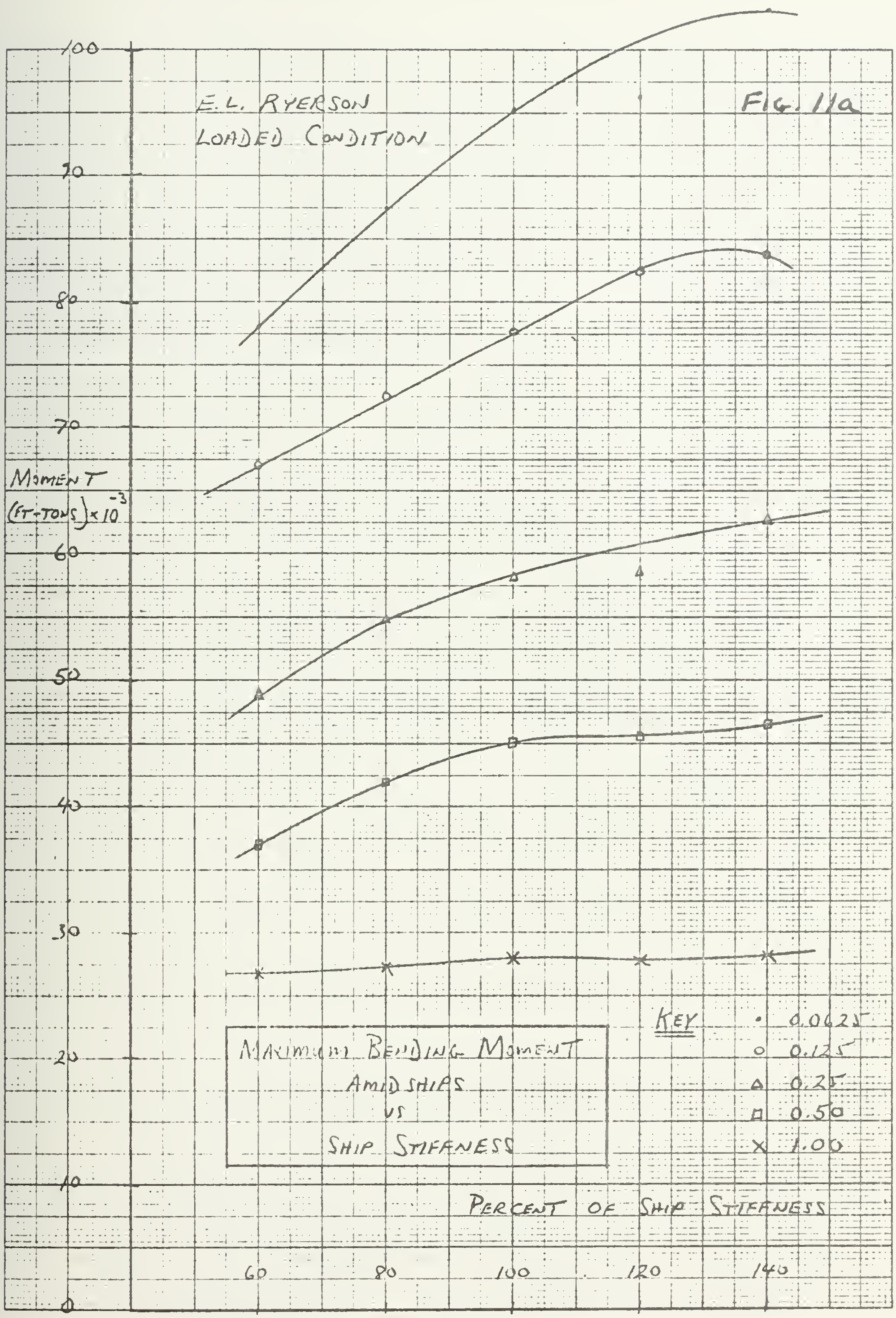
FIG. 9b

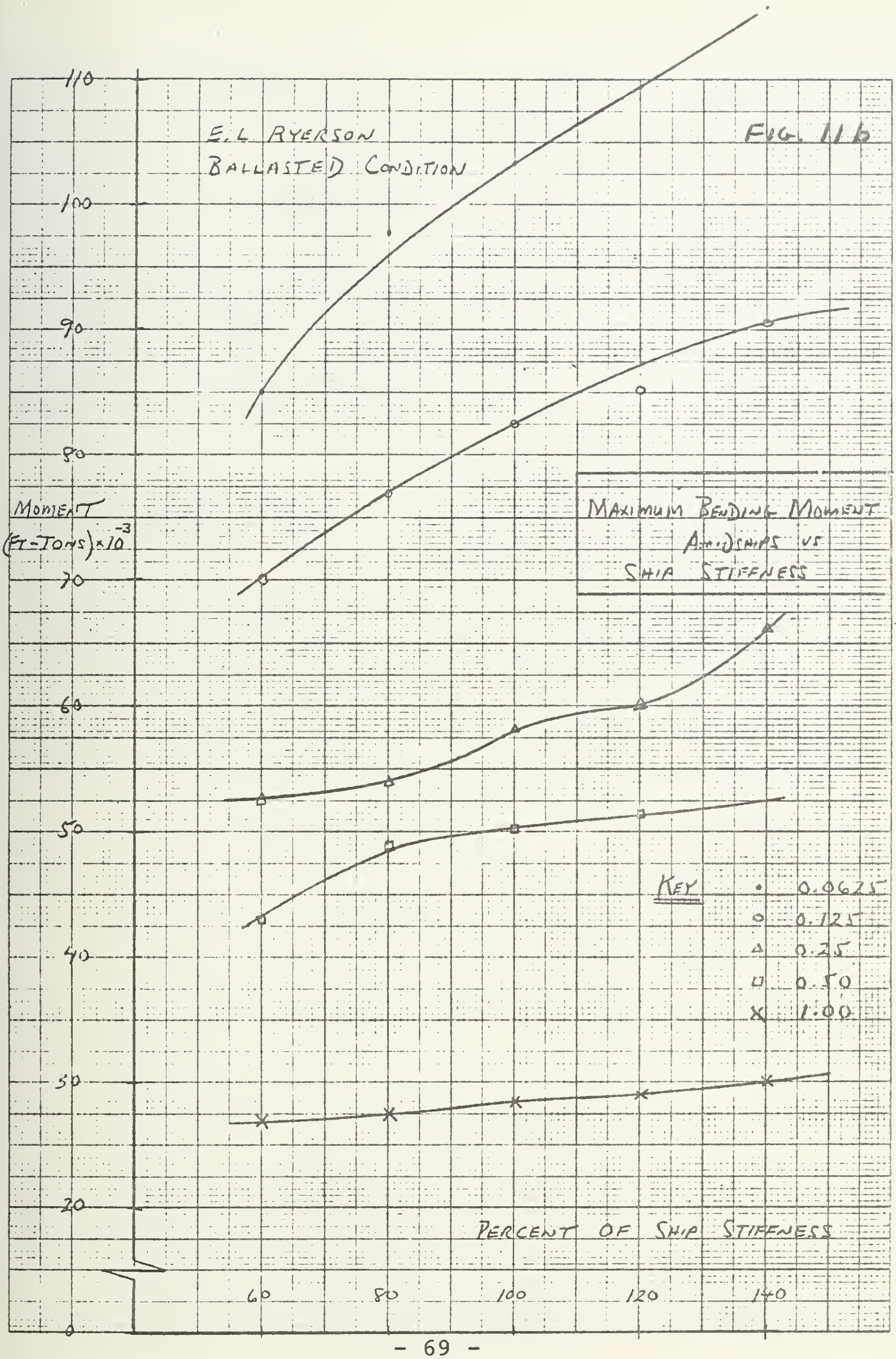

DECK STRESS
VS
SLAM DURATION
S.S MICHIGAN
BALLASTED CONDITION
DECK STRESS
(TONS / IN^2)
3.0
2.0
1.0
0
0.50
SLAM DURATION (SECS)
60 %
80 %
100 %
120 %
140 %



FOTINI - L
LOADED CONDITION
FIG. 10a
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
MOMENT (FT-TONS)×10⁻³
100
90
80
70
60
50
40
30
20
10
0
KEY
• 0.0625
○ 0.125
△ 0.25
□ 0.50
× 1.00
PERCENT OF SHIP STIFFNESS
60
80
100
120
140


FOTINI - L
BALLASTED CONDITION
FIG. 10 b
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
MOMENT
(FT-TONS) × 10^-3
100
90
80
70
60
50
40
30
20
10
0
KEY
• 0.0625
o 0.125
Δ 0.25
□ 0.50
× 1.00
PERCENT OF SHIP STIFFNESS
60
80
100
120
140


E.L. RYERSON
LOADED CONDITION
FIG. 11a
MOMENT (FT-TONS) x 10^-3
100
90
80
70
60
50
40
30
20
10
0
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
KEY
0.0625
0.125
0.25
0.50
1.00
PERCENT OF SHIP STIFFNESS
60
80
100
120
140


E. L. RYERSON
BALLASTED CONDITION
FIG. 11b
MAXIMUM BENDING MOMENT AMIDSHIPS VS SHIP STIFFNESS
MOMENT (FT-TONS)×10^-3
110
100
90
80
70
60
50
40
30
20
0
KEY
• 0.0625
o 0.125
Δ 0.25
□ 0.50
X 1.00
PERCENT OF SHIP STIFFNESS
60
80
100
120
140

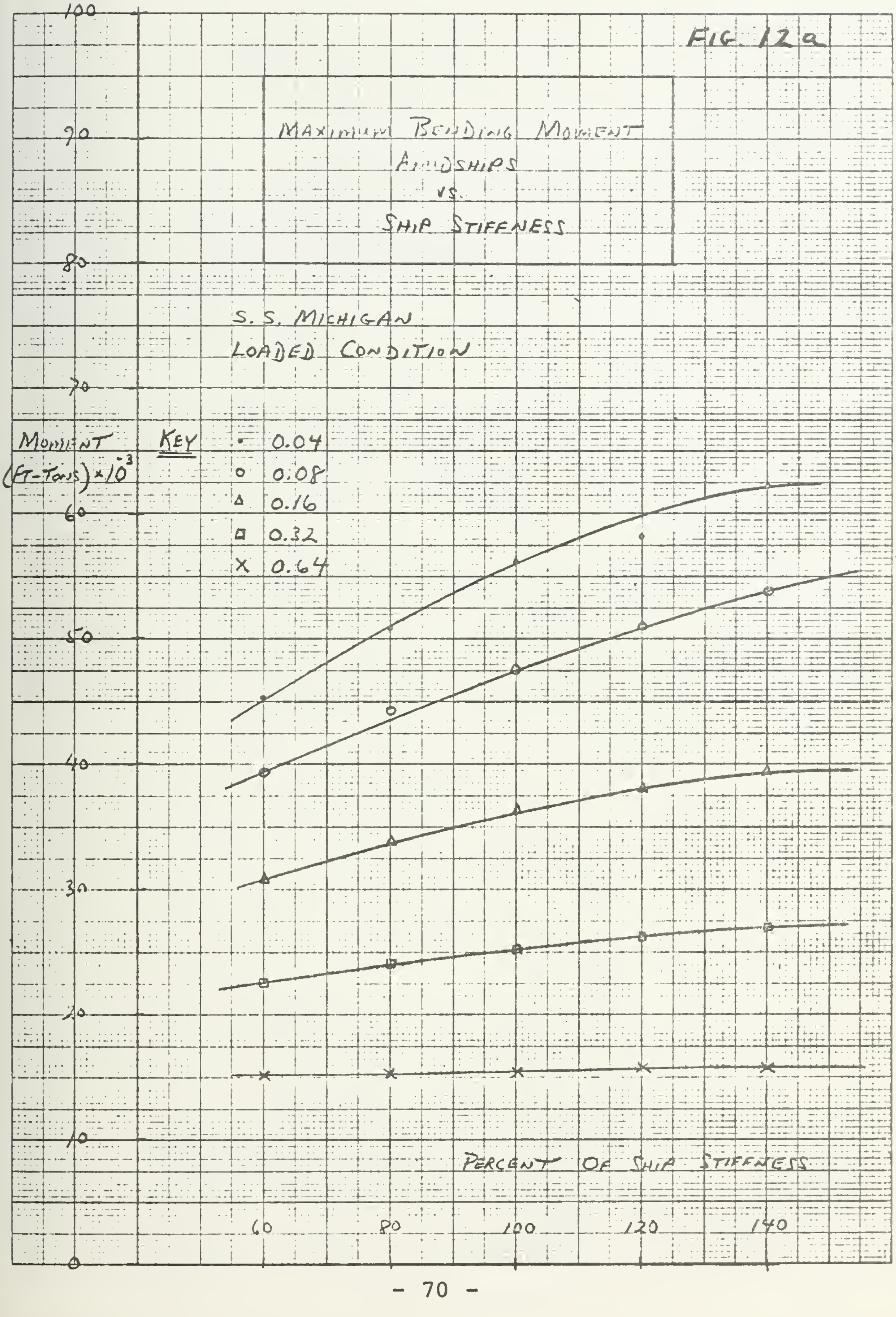


FIG. 12a
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS.
SHIP STIFFNESS
S. S. MICHIGAN
LOADED CONDITION
MOMENT
(FT-TONS) × 10^{-3}
KEY
• 0.04
○ 0.08
△ 0.16
□ 0.32
× 0.64
PERCENT OF SHIP STIFFNESS
100
90
80
70
60
50
40
30
20
10
0
60
80
100
120
140

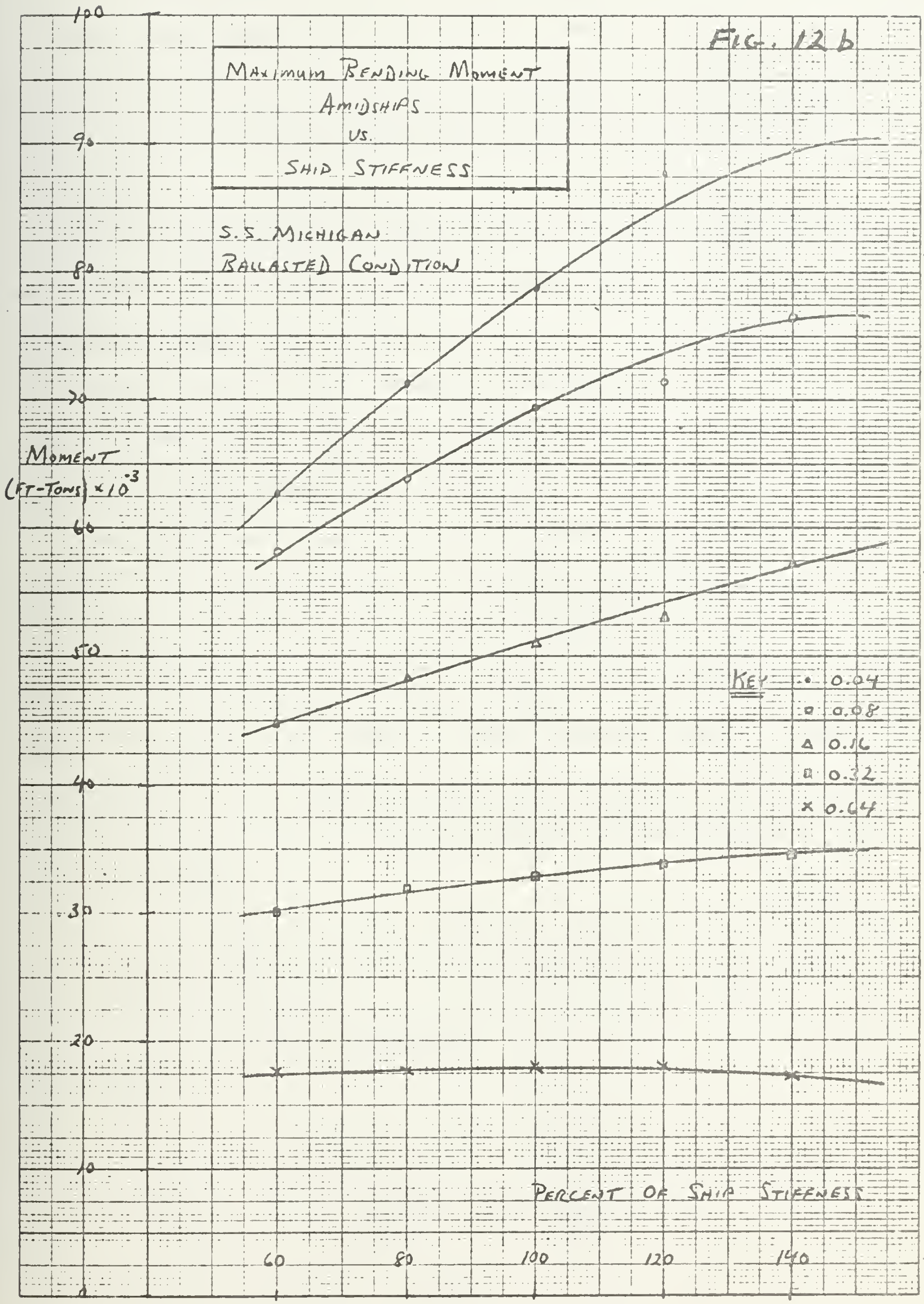


FIG. 12 b
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS.
SHIP STIFFNESS
S.S. MICHIGAN
BALLASTED CONDITION
MOMENT
(FT-TONS) × 10^-3
100
90
80
70
60
50
40
30
20
10
0
KEY
• 0.04
○ 0.08
△ 0.16
□ 0.32
× 0.64
PERCENT OF SHIP STIFFNESS
60
80
100
120
140

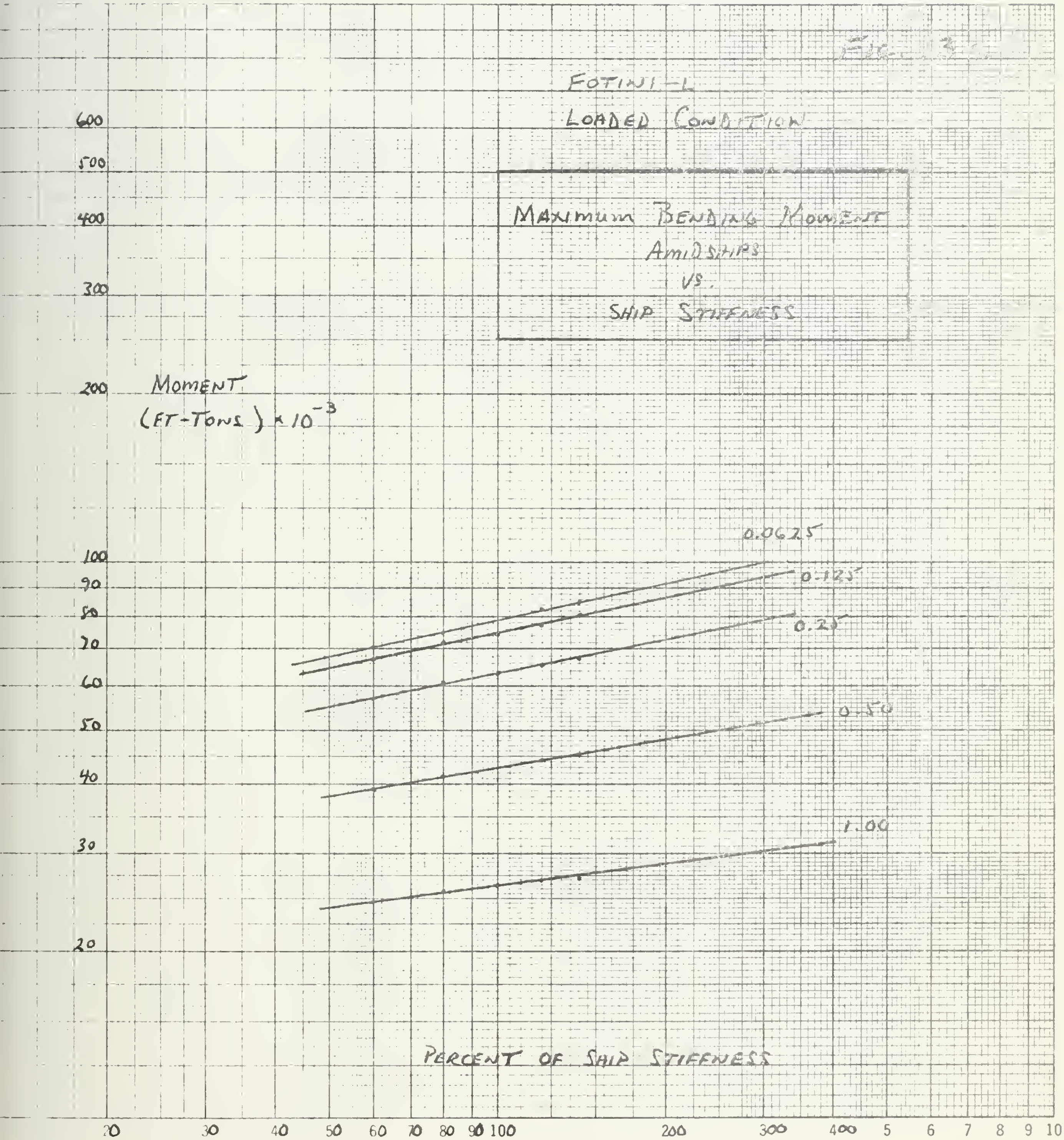
With this relation in mind, the slamming moment data were used to find the variables A, b, and C. These variables were then studied to see if they could be related to any of the parameters associated with the given ships or the given slamming conditions.

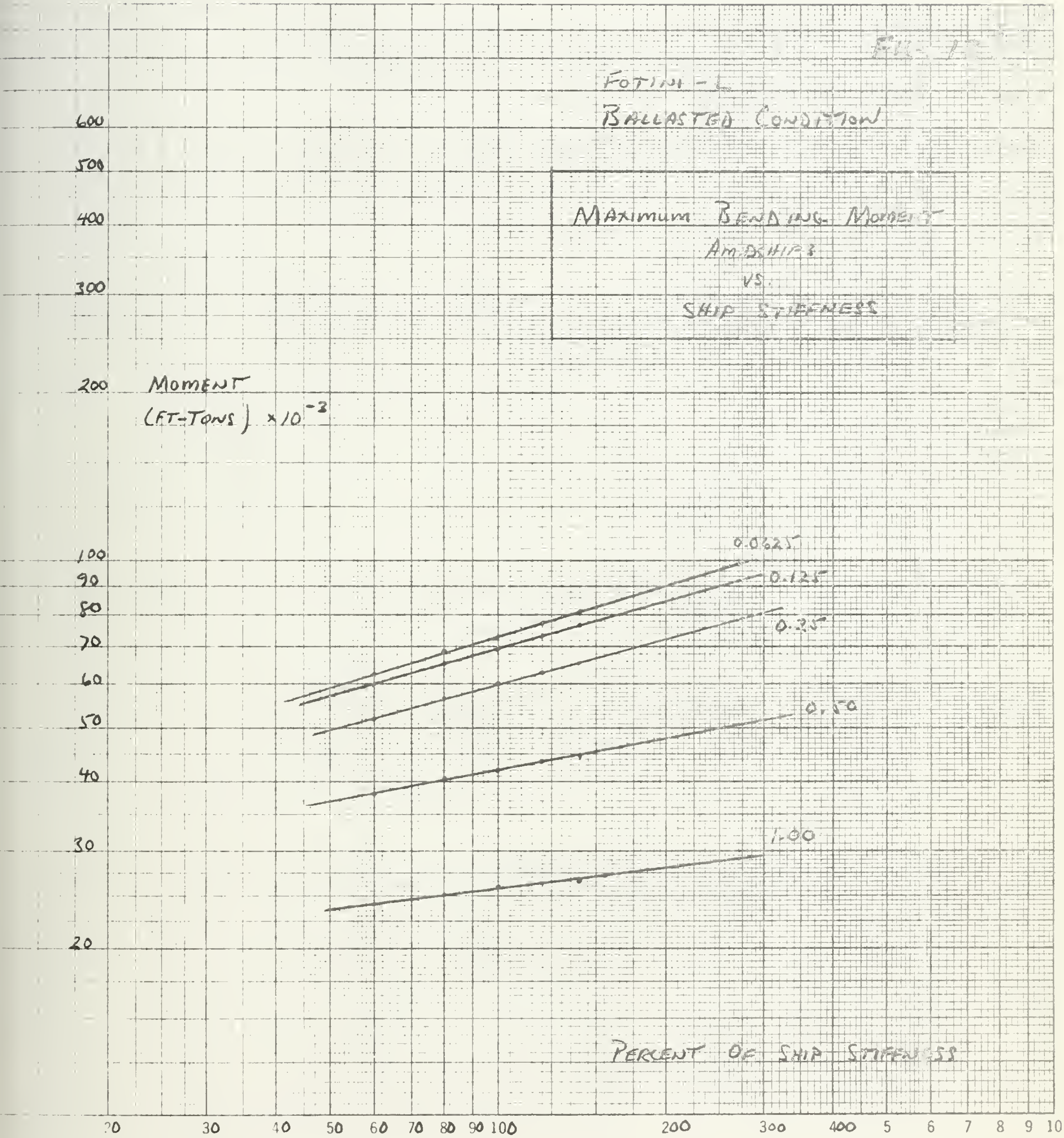
The bending moments were plotted against ship stiffness again, but this time on log-log graph paper. These plots are shown in Figures 13a, 13b, 14a, 14b, 15a, and 15b. For a given ship in either the loaded or ballasted condition and for each slam duration, the points approximated a straight line. Thus, the assumption of equation (27) seemed to be correct.

The lines were used to find values for A, b, and C in equation (27) for the ships in their loaded and ballasted conditions. The slope of a given line is the value of the exponent b for the corresponding slam duration and loading condition. Two points can then be read from the graph, substituted into equation (27), and the resulting equations solved simultaneously for A and C. This process is illustrated in the following example.

For this example, the FOTINI-L in the loaded condition and a slam duration of 0.0625 seconds will be used. Using Figure 13a the following points can be read from the upper most line.

Stiffness	Moment
60	70.5
140	84.9
200	92.0


FOTINI-L
LOADED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS.
SHIP STIFFNESS
MOMENT
(FT-TONS) x 10^-3
0.0625
0.125
0.25
0.50
1.00
PERCENT OF SHIP STIFFNESS
600
500
400
300
200
100
90
80
70
60
50
40
30
20
20
30
40
50
60
70
80
90
100
200
300
400
5
6
7
8
9
10


FOTINI - L
BALLASTED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
MOMENT
(FT-TONS) x10^-3
600
500
400
300
200
100
90
80
70
60
50
40
30
20
0.0625
0.125
0.25
0.50
1.00
PERCENT OF SHIP STIFFNESS
20
30
40
50
60
70
80
90
100
200
300
400
5
6
7
8
9
10

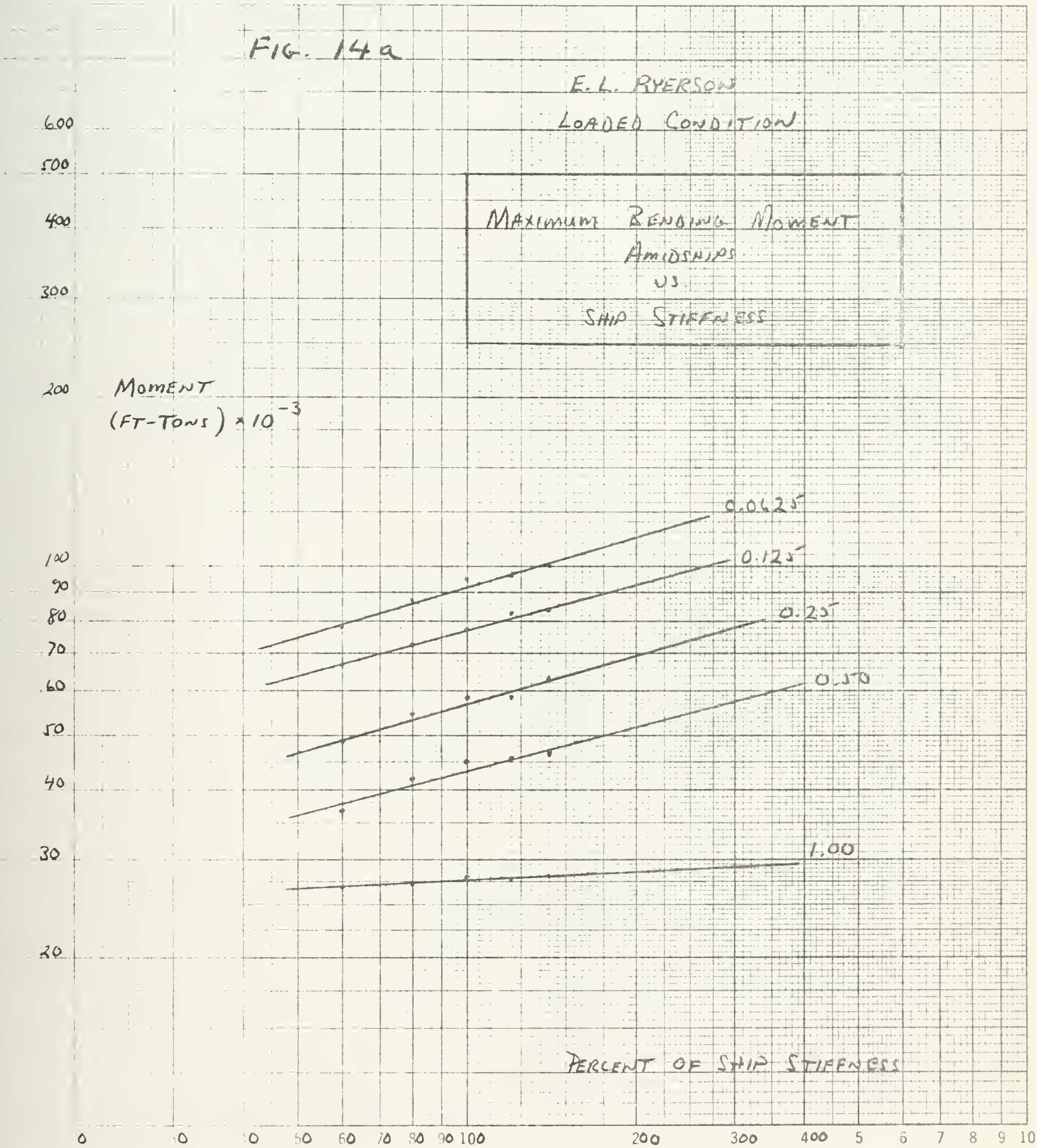


Fig. 14a
E. L. Ryerson
Loaded Condition
Maximum Bending Moment
Amidships
vs
Ship Stiffness
Moment
(Ft-Tons) × 10-3
0.0625
0.125
0.25
0.50
1.00
Percent of Ship Stiffness
600
500
400
300
200
100
90
80
70
60
50
40
30
20
50 60 70 80 90 100 200 300 400 5 6 7 8 9 10

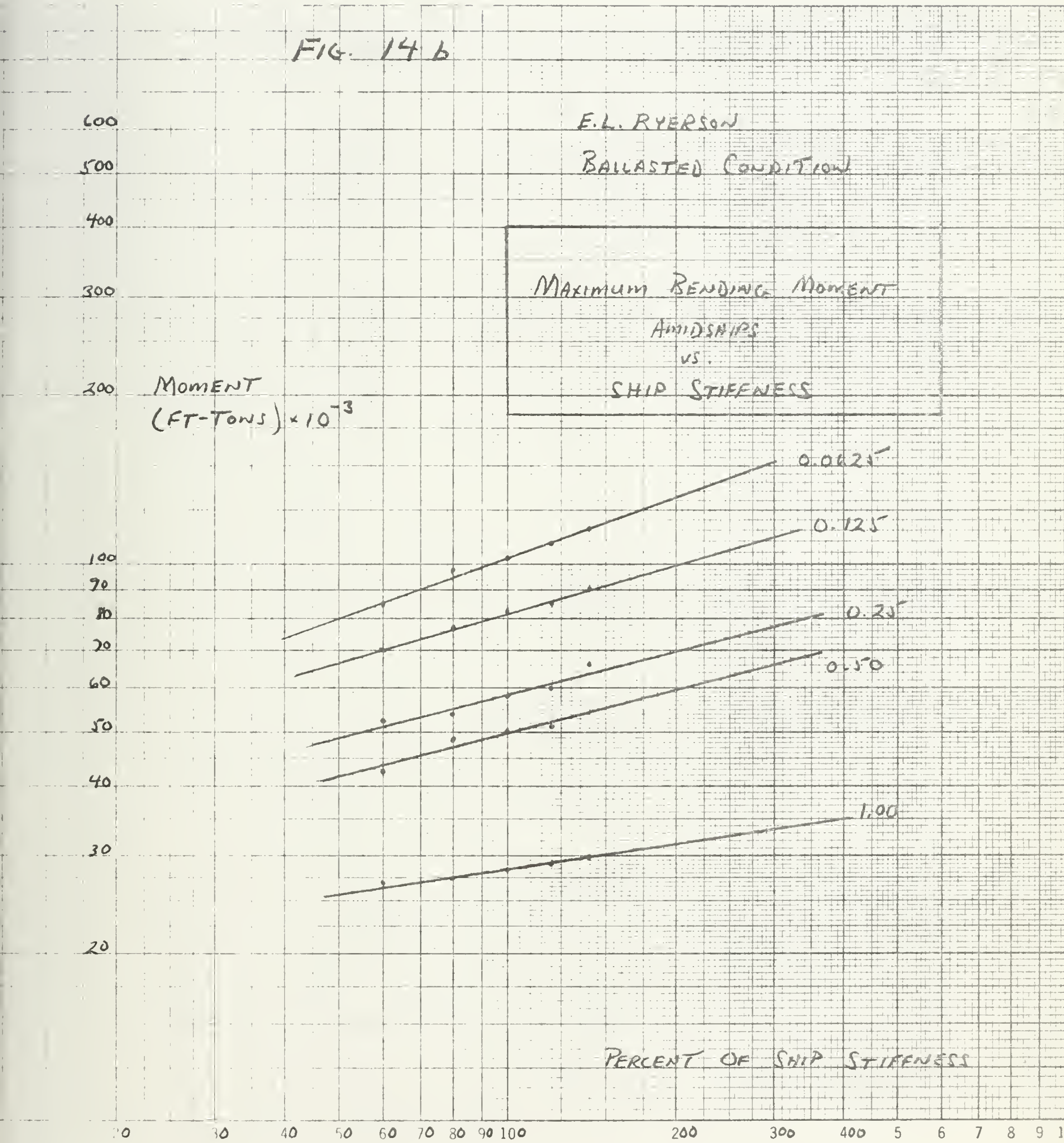
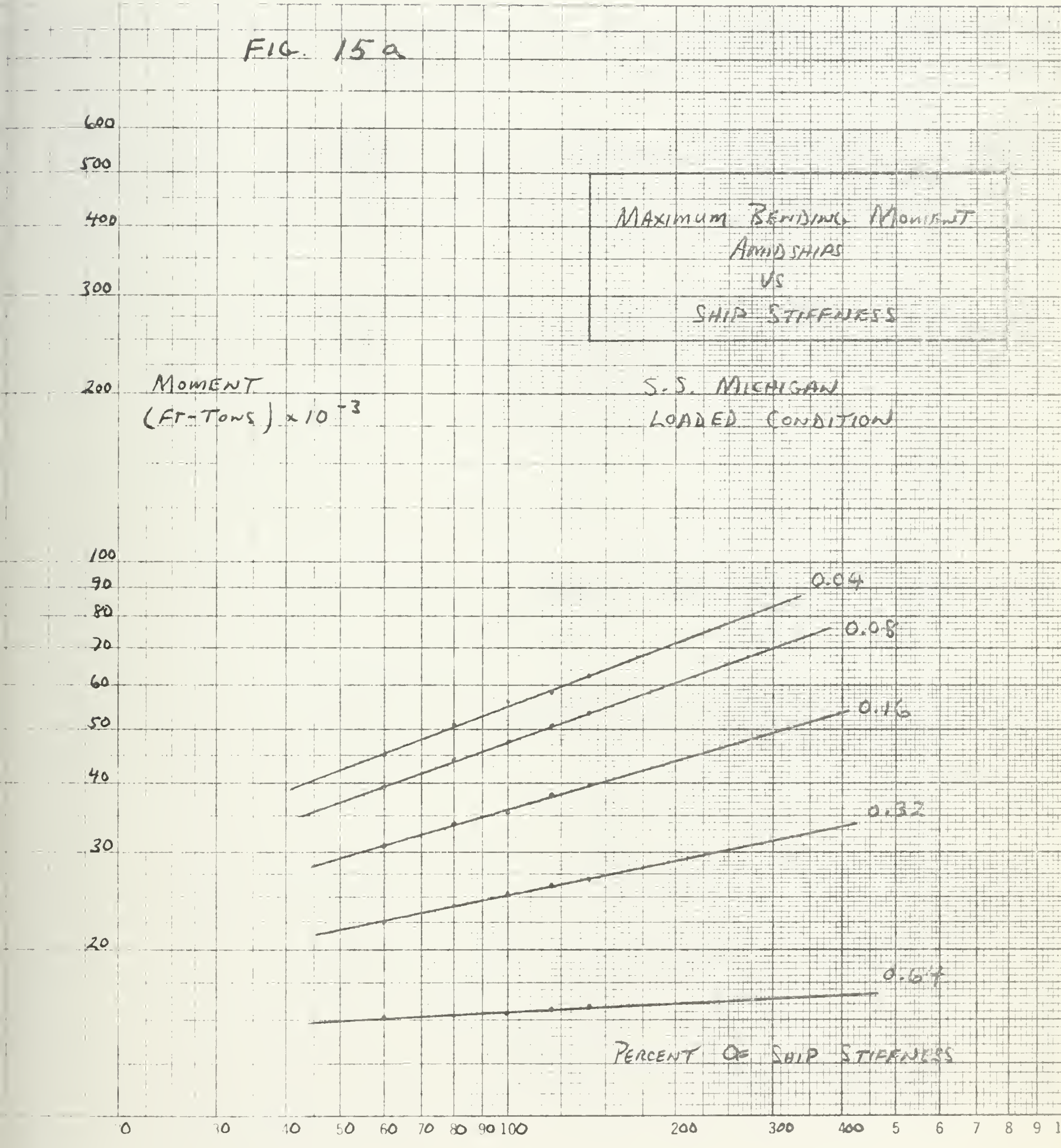


FIG. 14 b
E.L. RYERSON
BALLASTED CONDITION
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS.
SHIP STIFFNESS
MOMENT
(FT-TONS) × 10^-3
0.0625
0.125
0.25
0.50
1.00
PERCENT OF SHIP STIFFNESS
600
500
400
300
200
100
90
80
70
60
50
40
30
20
20
30
40
50
60
70
80
90
100
200
300
400
5
6
7
8
9
10

FIG. 15a


MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
S.S. MICHIGAN
LOADED CONDITION
MOMENT
(FT-TONS) x 10^-3
PERCENT OF SHIP STIFFNESS
0.04
0.08
0.16
0.32
0.64
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
30
40
50
60
70
80
90
100
200
300
400
5
6
7
8
9
10

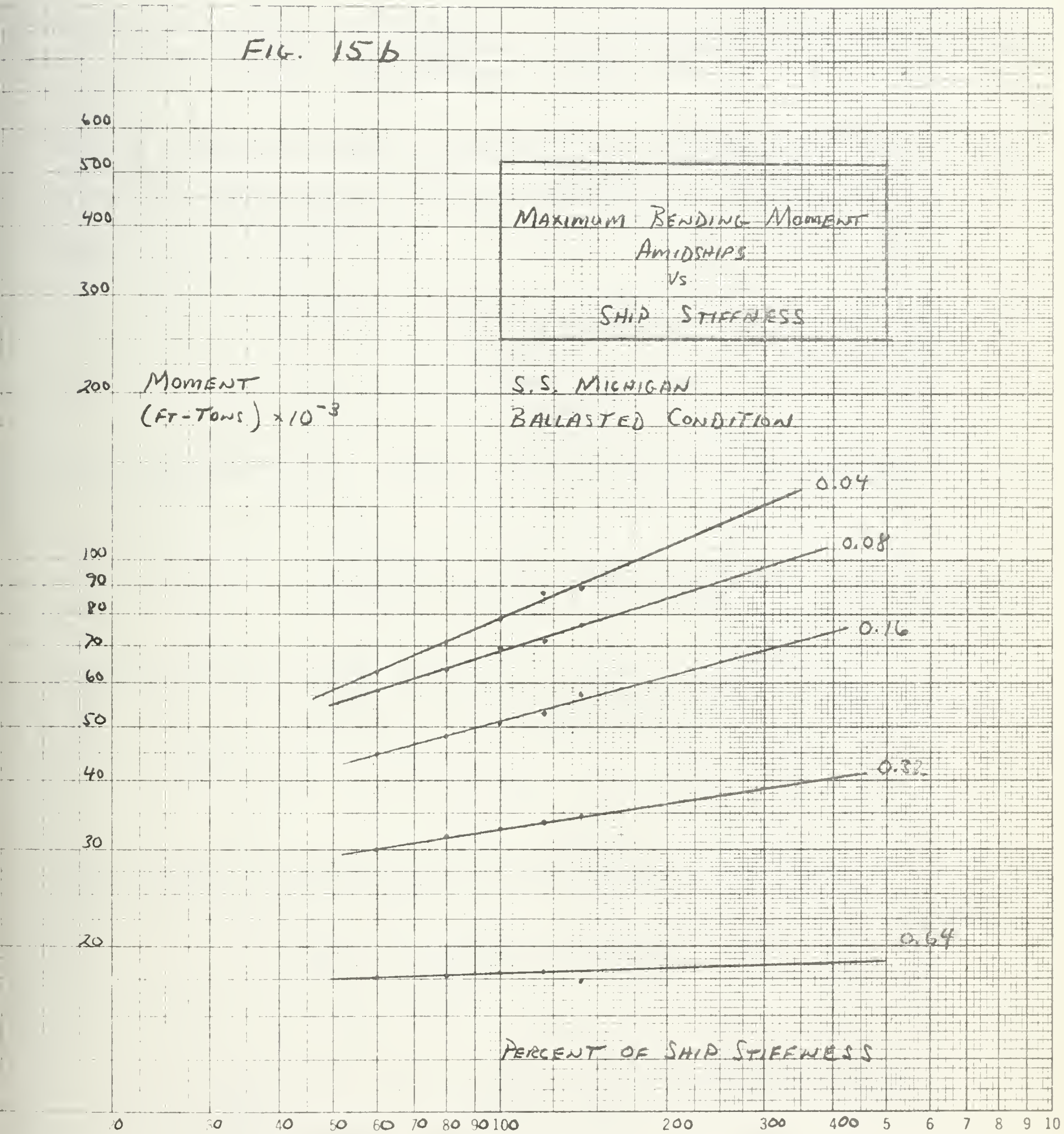


FIG. 15 b
MAXIMUM BENDING MOMENT
AMIDSHIPS
VS
SHIP STIFFNESS
S.S. MICHIGAN
BALLASTED CONDITION
MOMENT
(FT-TONS) × 10^-3
600
500
400
300
200
100
90
80
70
60
50
40
30
20
0.04
0.08
0.16
0.32
0.64
PERCENT OF SHIP STIFFNESS
40
50
60
70
80
90
100
200
300
400
5
6
7
8
9
10

The slope is then calculated from the two extreme points as follows:

$$\text{slope} = \frac{\text{change in moment}}{\text{change in stiffness}} = \frac{92.0 - 70.5}{200 - 60} = 0.154$$

This is the value of the exponent b. Taking the first two points and substituting them into equation (27) gives two equations in two unknowns.

$$84.9 = A(140)^{0.154} + C \qquad (28)$$

$$70.5 = A(60)^{0.154} + C$$

From these equations, A and C can be solved for as follows:

$$84.9 = 2.14\ A + C$$

$$\underline{70.5 = 1.88\ A + C}$$

subtracting the bottom equation from the top equation,

$$14.4 = 0.26\ A$$

$$\therefore \quad A = 55.4$$

Substituting A back into equation (28) and solving for C,

$$84.9 = 55.4(140)^{0.154} + C$$

$$\therefore \quad C = -33.7$$

This calculation was done for the three ships in both the loaded and ballasted condition and for all slam durations.

The results are tabulated in Tables 10, 11, and 12. It was found that the value of C is always negative; so for simplicity, the negative sign was dropped and equation (27) changed to:

$$\text{Slam Moment} = A(EI)^{b} - C \tag{29}$$

It should be noted that EI is the percentage, i.e. 60, 80, etc. and the left hand side of equation (29) must be multiplied by 10^3 to get the true moment in tons-feet.

The values of the coefficient A, the exponent b, and the constant C for equation (29) are plotted against slam duration for each ship in both loading conditions. The graphs for the exponent b are shown in Figures 16a, 17a, and 18a. The plots of coefficient A are found in Figures 16b, 17b, and 18b. Finally, Figures 16c, 17c, and 18c illustrate the plots of the constant C.

It can be seen from these plots that for each individual ship, the points for loaded and ballasted conditions follow the same curve fairly closely. The curves for b and A were drawn in closer to the higher values calculated so as to predict a slam moment closer to the greater true moment obtained from the computer. For the same reason the curve for C was drawn closer to the minimum for each ship. Since it is negative in equation (29) this will also allow a larger moment to be predicted.

Given these graphs along with equation (29), for any of the

TABLE 10

Exponent b for Equation (29)

Ship and Loading Condition	Slam Duration (sec)				
	0.0625	0.125	0.25	0.5	1.0
FOTINI-L, Loaded	0.154	0.136	0.112	0.064	0.030
FOTINI-L, Ballasted	0.196	0.176	0.144	0.069	0.027
E.L. RYERSON, Loaded	0.239	0.186	0.144	0.099	0.012
E.L. RYERSON, Ballasted	0.334	0.211	0.131	0.112	0.032

	Slam Duration (sec)				
	0.04	0.08	0.16	0.32	0.64
MICHIGAN, Loaded	0.189	0.154	0.093	0.045	0.007
MICHIGAN, Ballasted	0.307	0.199	0.123	0.045	0.005

TABLE 11

Coefficient A for Equation (29)

Ship and Loading Condition	Slam Duration (sec) 0.0625	0.125	0.25	0.5	1.0
FOTINI-L, Loaded	55.4	61.4	66.9	90.0	83.3
FOTINI-L, Ballasted	44.0	50.9	55.8	83.8	135.0
E.L. RYERSON, Loaded	38.3	47.3	57.9	70.8	120.0
E.L. RYERSON, Ballasted	24.3	42.6	64.0	72.5	120.0

	Slam Duration (sec) 0.04	0.08	0.16	0.32	0.64
MICHIGAN, Loaded	45.7	55.4	71.7	84.0	80.0
MICHIGAN, Ballasted	26.7	44.9	58.9	90.0	125.0

TABLE 12

Constant C for Equation (29)

Ship and Loading Condition	Slam Duration (sec) 0.0625	0.125	0.25	0.5	1.0
FOTINI-L, Loaded	33.7	39.8	48.4	78.0	69.5
FOTINI-L, Ballasted	35.8	44.7	48.3	72.8	127.4
E.L. RYERSON, Loaded	22.8	34.1	55.0	68.4	99.2
E.L. RYERSON, Ballasted	10.3	30.8	58.3	70.7	110.6

	Slam Duration (sec) 0.04	0.08	0.16	0.32	0.64
MICHIGAN, Loaded	54.1	64.8	73.9	78.0	67.0
MICHIGAN, Ballasted	30.7	43.5	52.2	77.9	110.0

FIG. 16a

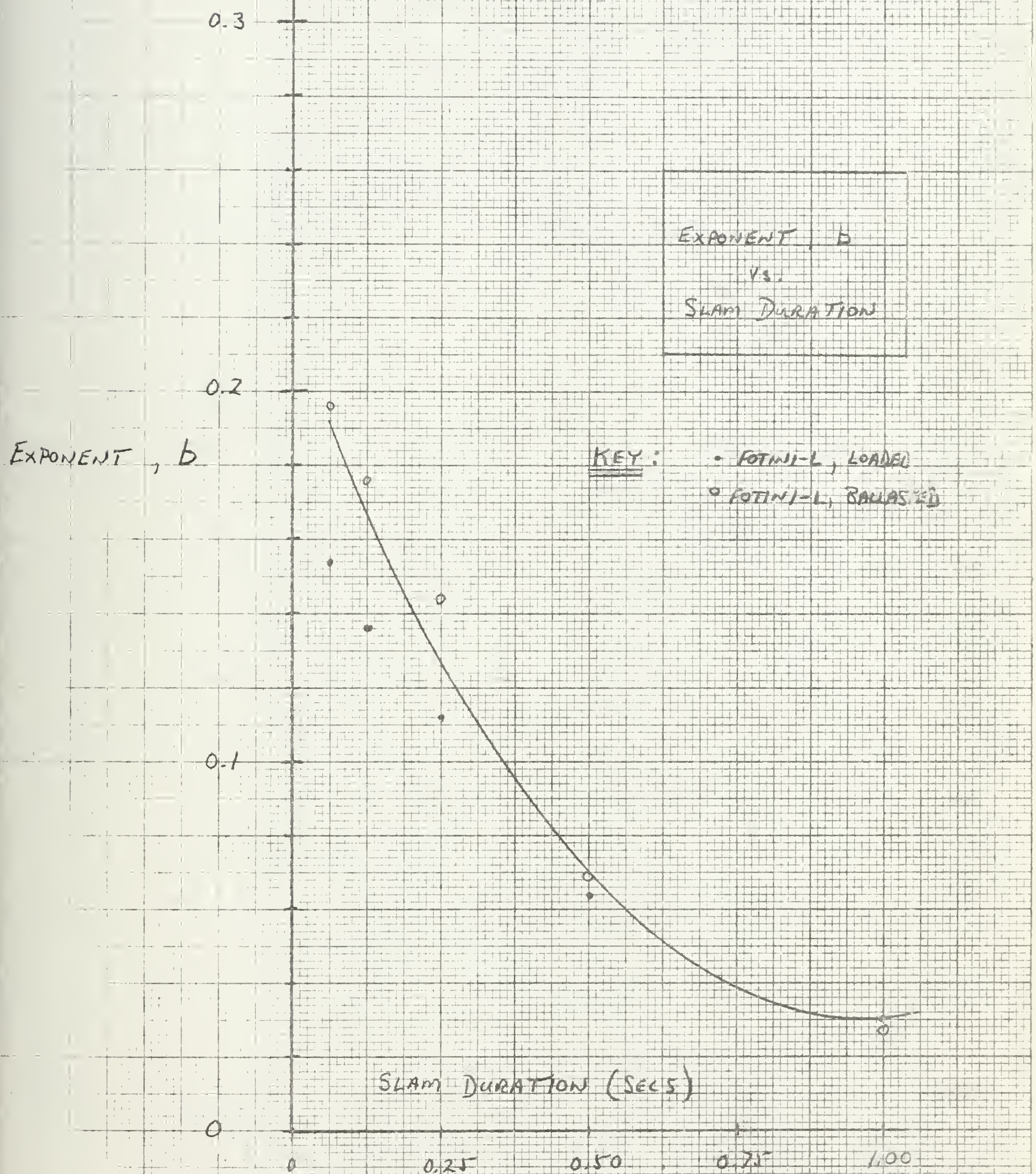
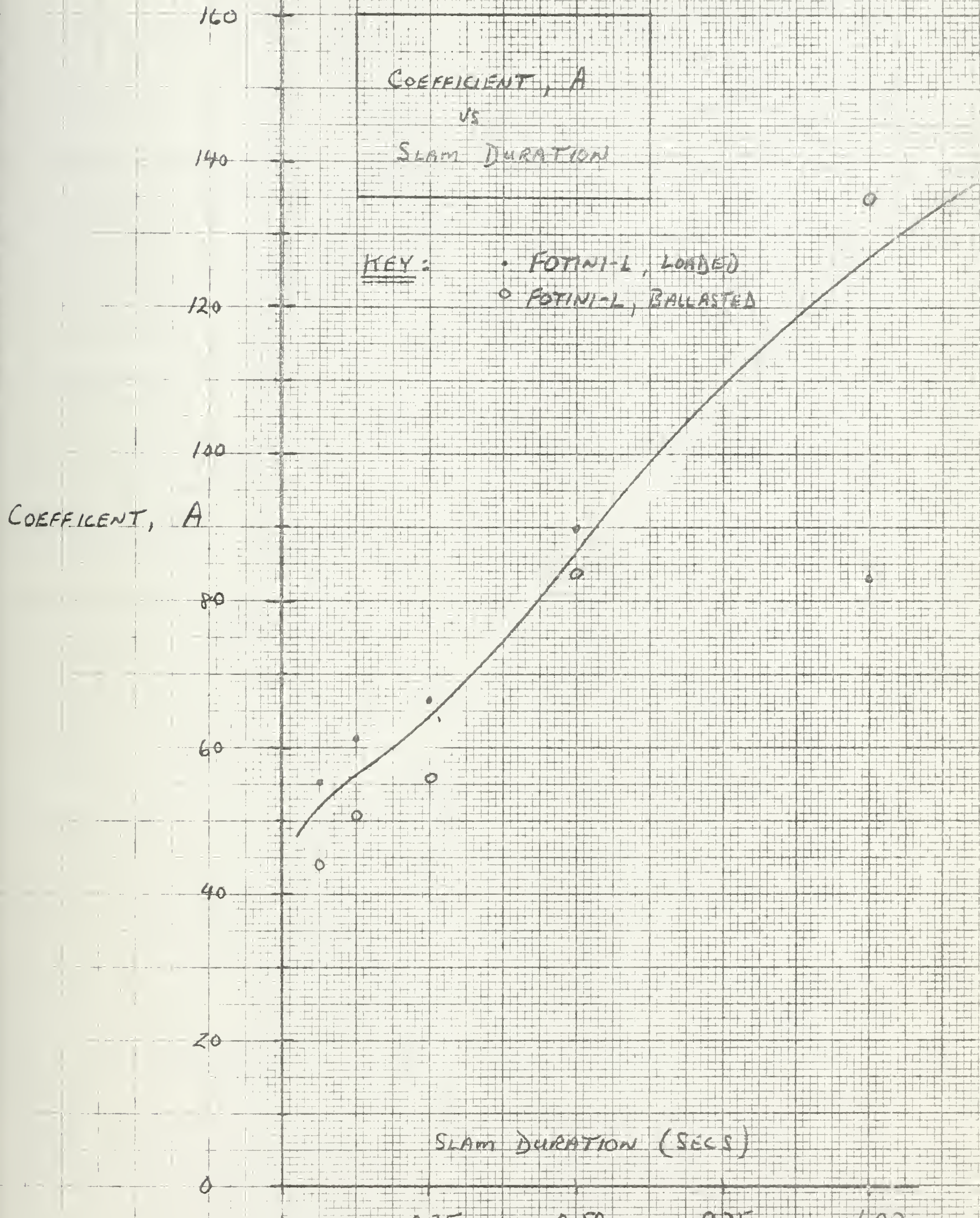

EXPONENT, b
VS.
SLAM DURATION
KEY:
• FOTINI-L, LOADED
○ FOTINI-L, BALLASTED
EXPONENT, b
0.3
0.2
0.1
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

FIG. 16 b


COEFFICIENT, A
VS
SLAM DURATION
KEY:
· FOTINI-L, LOADED
○ FOTINI-L, BALLASTED
160
140
120
100
80
60
40
20
0
COEFFICENT, A
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

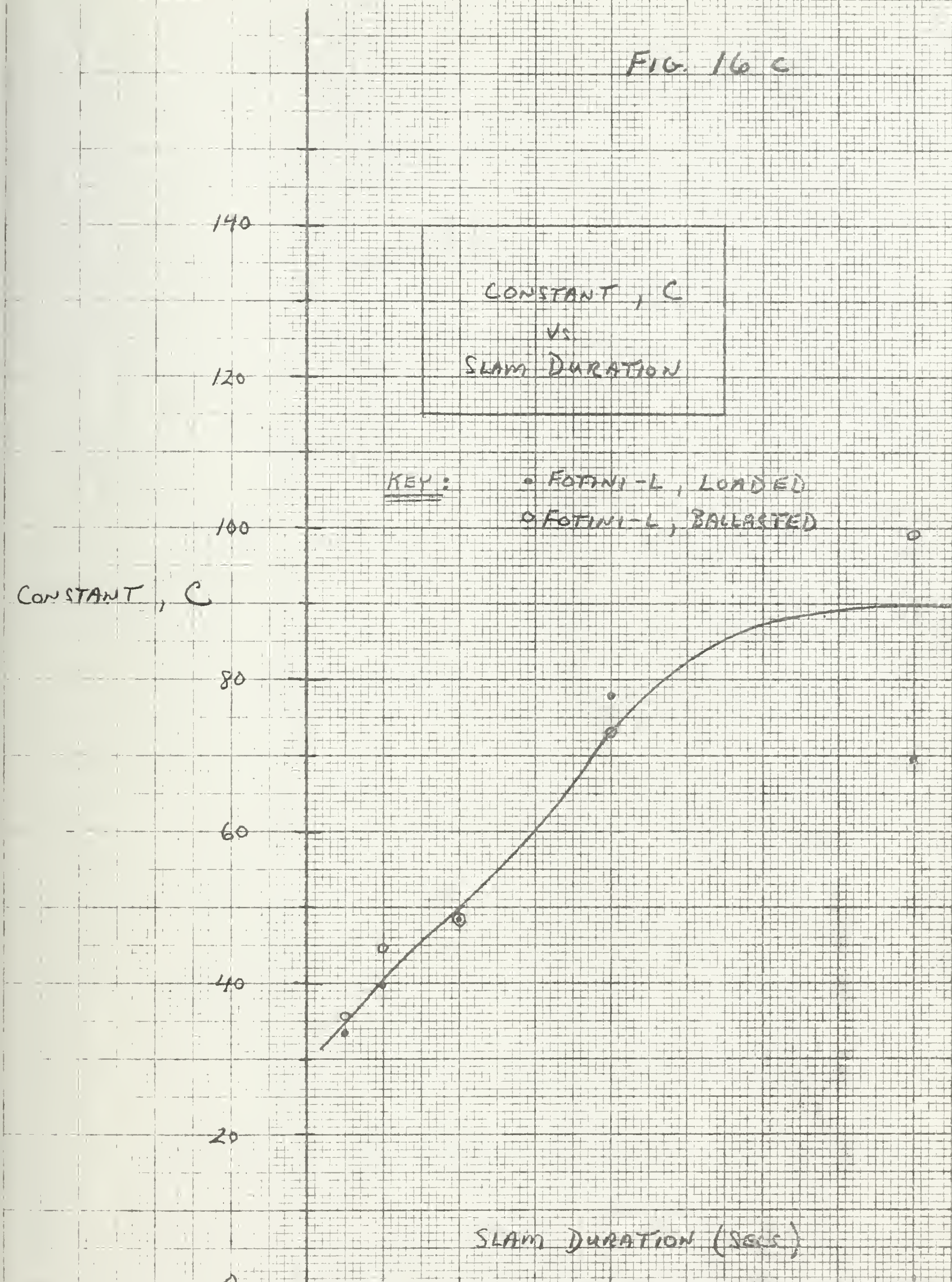


FIG. 16 C
CONSTANT, C
VS
SLAM DURATION
KEY:
• FOTINI-L, LOADED
⊙ FOTINI-L, BALLASTED
CONSTANT, C
140
120
100
80
60
40
20
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

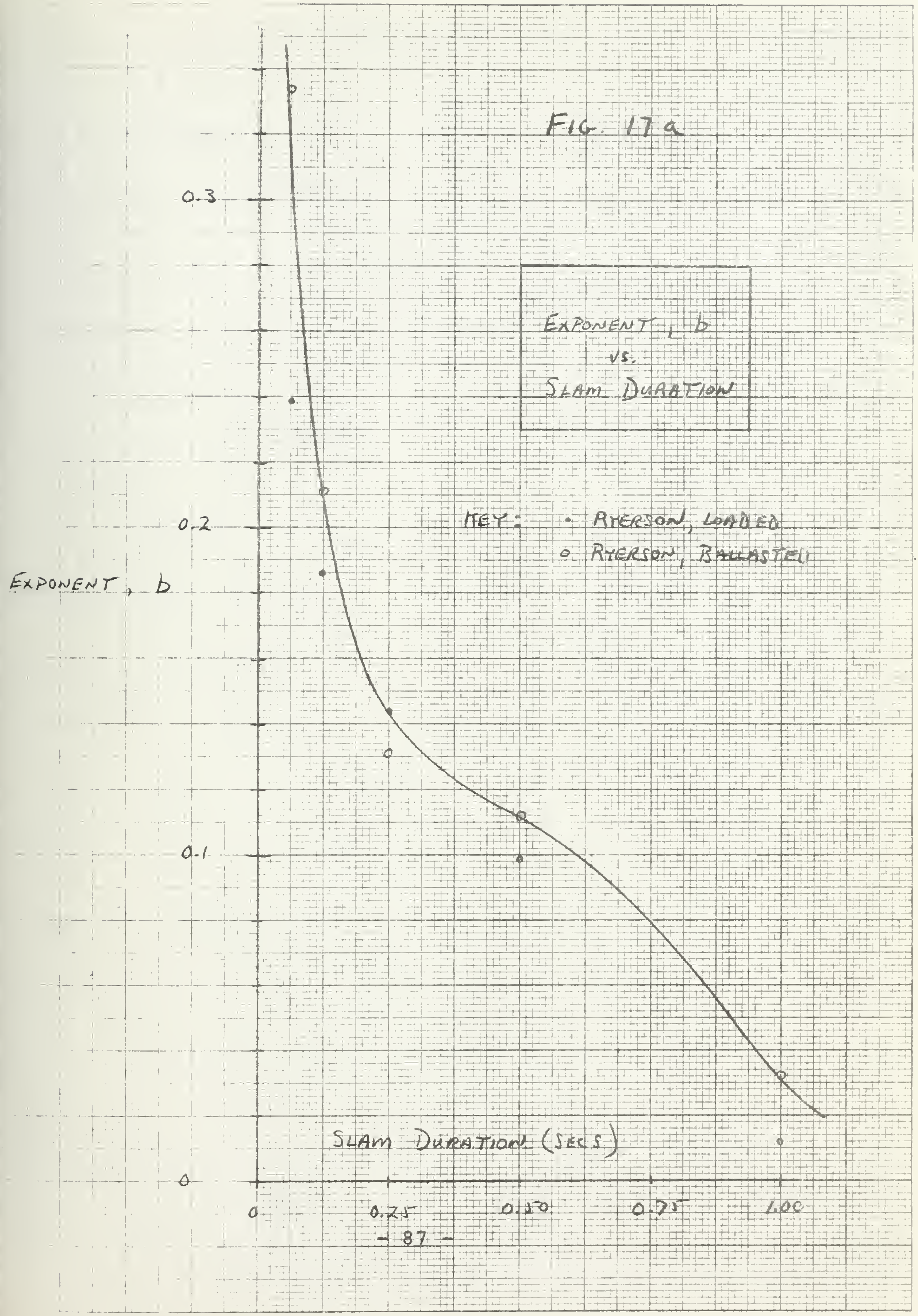


FIG. 17a
EXPONENT, b
VS.
SLAM DURATION
KEY: • RYERSON, LOADED
o RYERSON, BALLASTED
EXPONENT, b
0.3
0.2
0.1
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

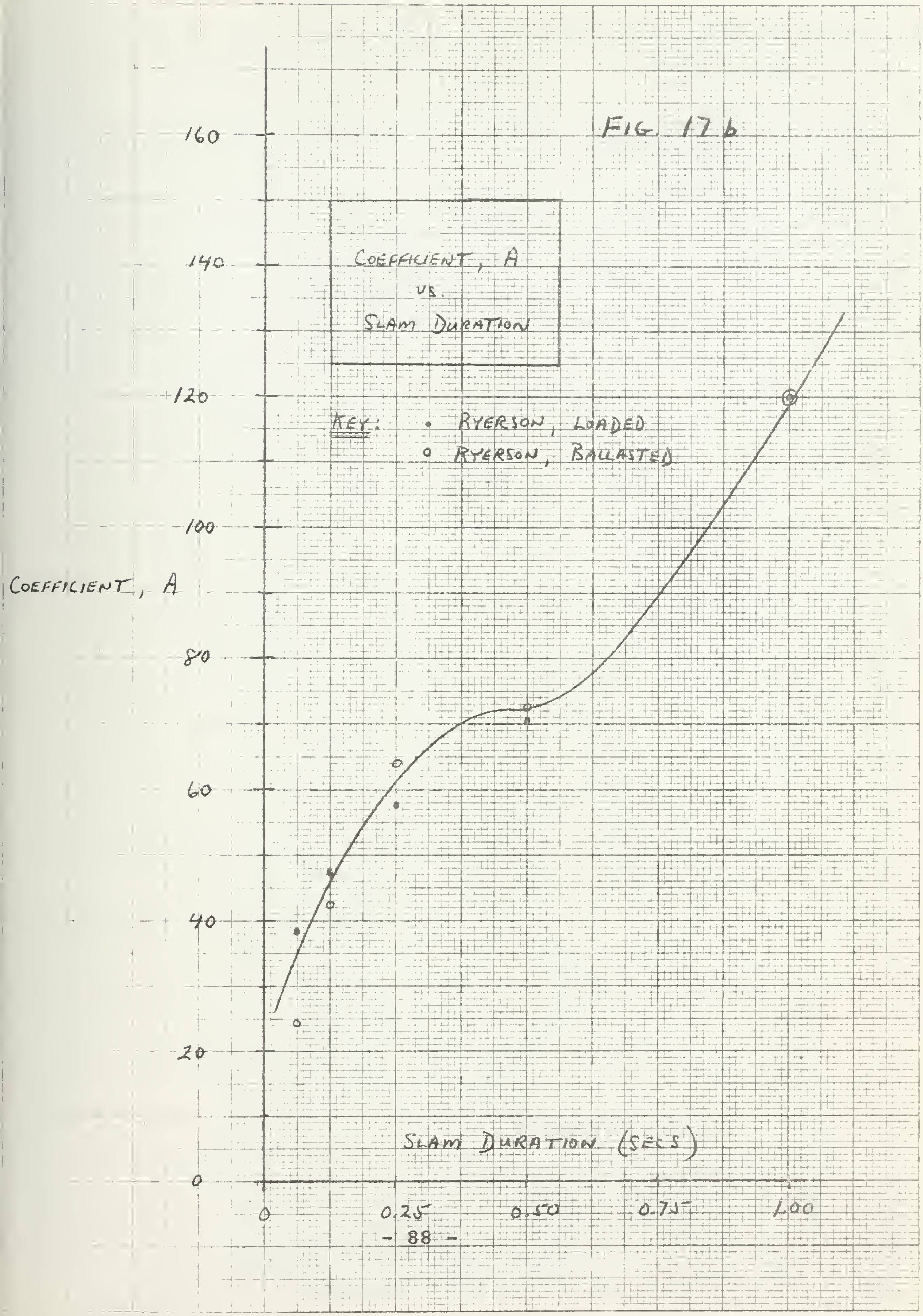
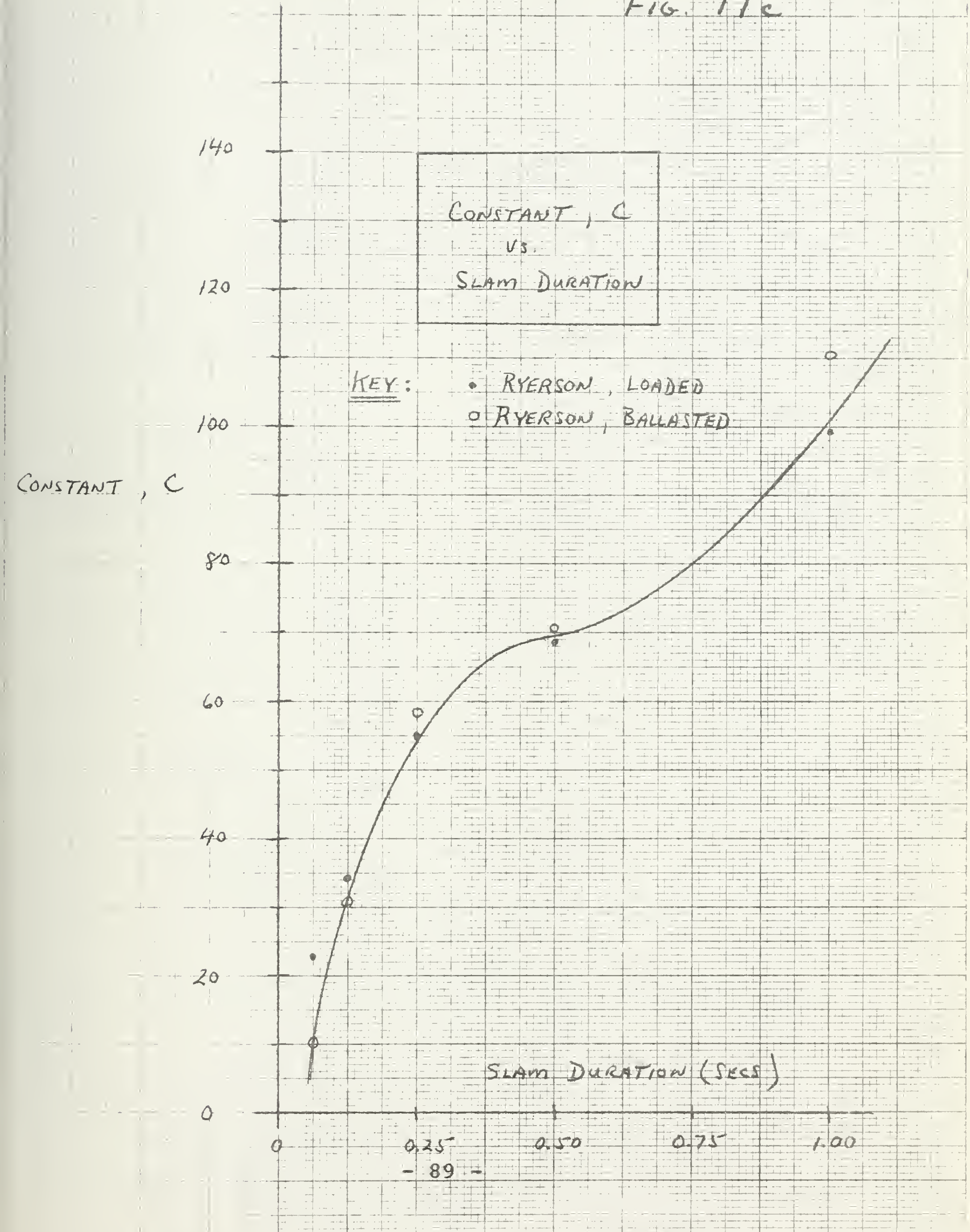


FIG. 17 b
COEFFICIENT, A
VS.
SLAM DURATION
KEY: • RYERSON, LOADED
o RYERSON, BALLASTED
COEFFICIENT, A
160
140
120
100
80
60
40
20
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

FIG. 17c


CONSTANT, C
VS.
SLAM DURATION
KEY:
• RYERSON, LOADED
o RYERSON, BALLASTED
CONSTANT, C
140
120
100
80
60
40
20
0
SLAM DURATION (SECS)
0
0.25
0.50
0.75
1.00

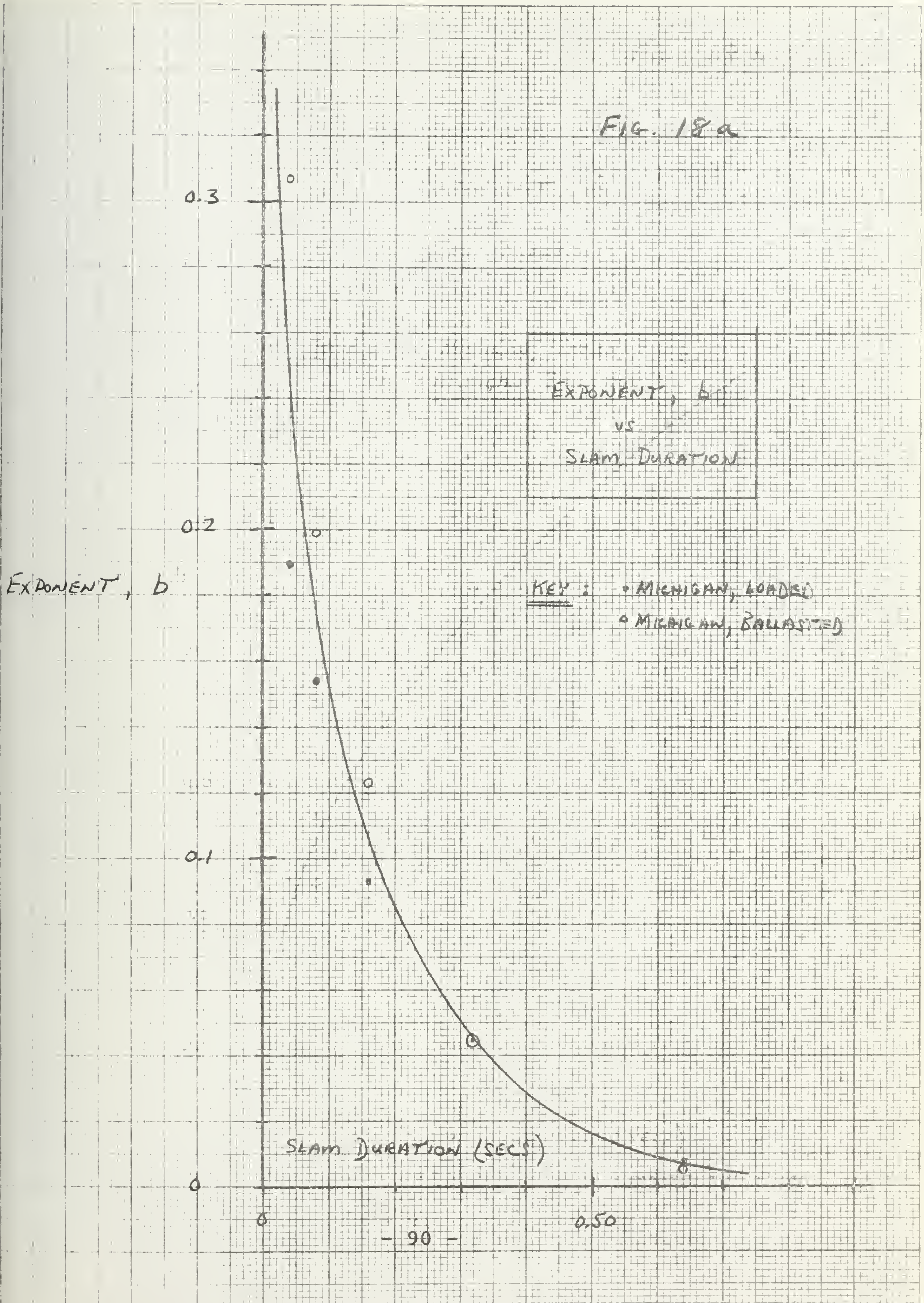


FIG. 18a
EXPONENT, b
vs
SLAM DURATION
KEY : • MICHIGAN, LOADED
∘ MICHIGAN, BALLASTED
EXPONENT, b
0.3
0.2
0.1
0
SLAM DURATION (SECS)
0
0.50

FIG. 18 b

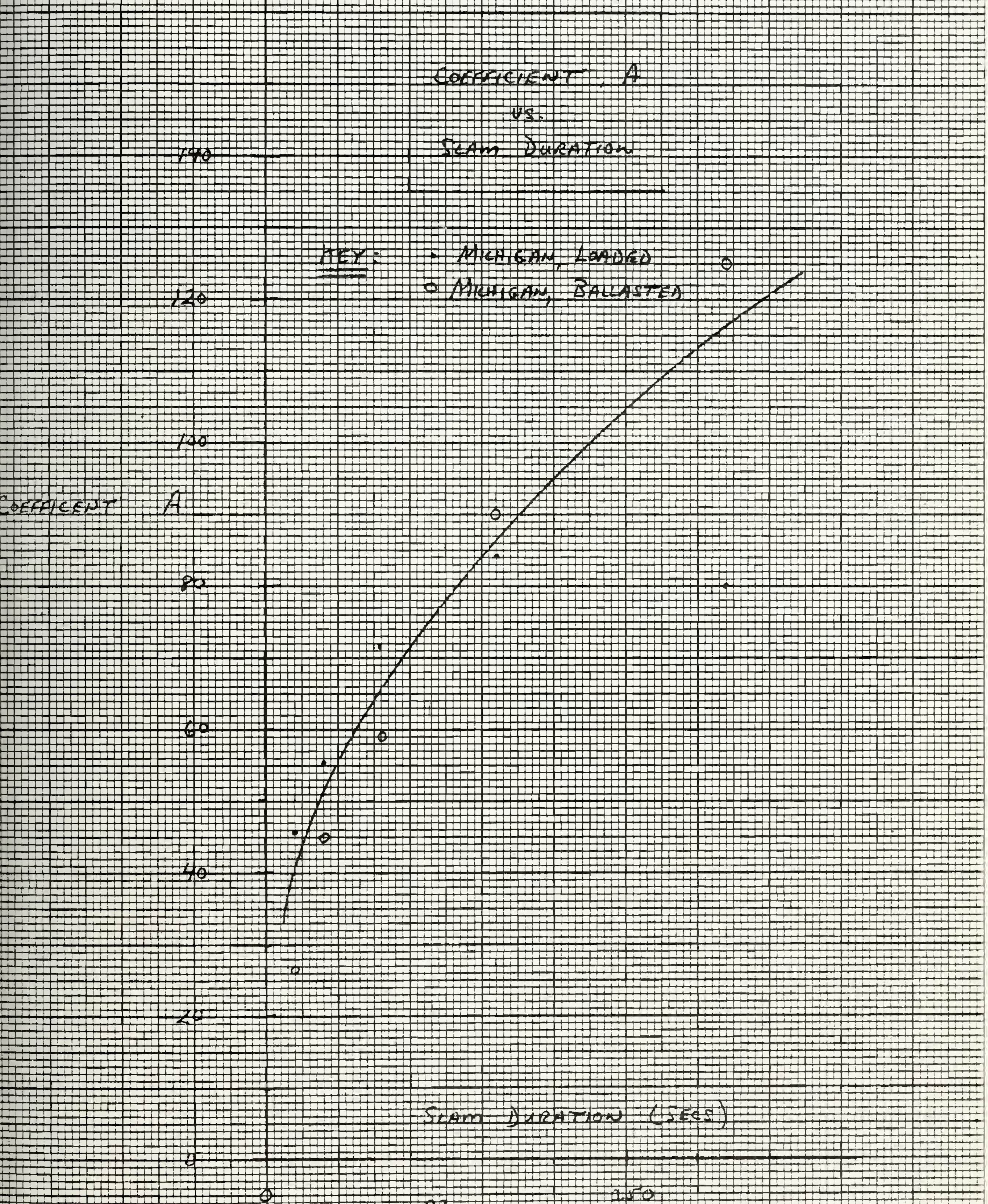
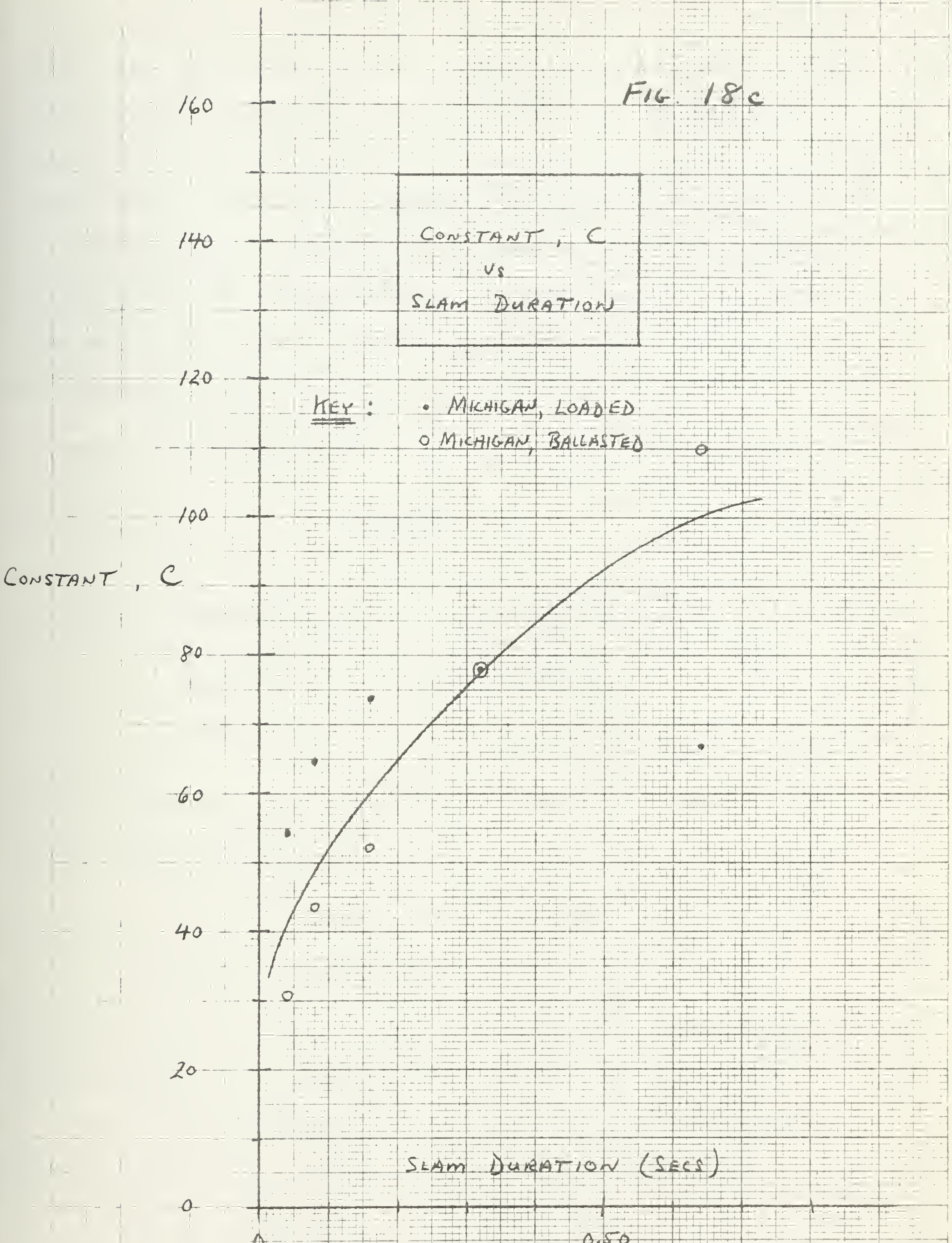

COEFFICIENT A
VS.
SLAM DURATION
KEY:
• MICHIGAN, LOADED
○ MICHIGAN, BALLASTED
140
120
100
80
60
40
20
0
COEFFICENT A
SLAM DURATION (SECS)
0
150

FIG. 18c


CONSTANT, C
VS
SLAM DURATION
KEY: • MICHIGAN, LOADED
o MICHIGAN, BALLASTED
CONSTANT, C
160
140
120
100
80
60
40
20
0
SLAM DURATION (SECS)
0
0.50

three ships discussed, in any loading condition, and for any value of hull stiffness, the maximum slamming moment amidships can be found for a known slam duration. As an example, suppose the maximum bending moment due to slamming for the FOTINI-L with bending stiffness 60% of the original design value and a slam duration of 0.25 seconds (half-sine impulse of 100 ton-seconds) is required. From Figures 16 the values for b, A, and C are read and tabulated below.

FOTINI-L, Slam Duration = 0.25

$$b = 0.126$$
$$A = 64.0$$
$$C = 50.0$$

Substituting these values into equation (29) gives

$$\text{Slam Moment} = 64.0(60)^{0.126} - 50.0$$

$$\text{Slam Moment} = 57.2 \times 10^{3} \text{ tons feet}$$

From the computer results (Table 3), the true moments for these conditions are:

FOTINI-L, Loaded - 57.3×10^{3} tons feet

FOTINI-L, Ballasted - 52.0×10^{3} tons feet

The predicted moment from equation (29) falls between the two actual moments, but closer to the higher one, as expected.

The stress predicted by equation (29) is found by dividing the slam moment by the reduced section modulus read from

Table 6.

$$\sigma_{Deck} = \frac{57.2 \times 10^3 \text{ tons-feet}}{95{,}133.6 \text{ in.}^2\text{-feet}} = 0.601 \text{ tons/in.}^2$$

The true stresses from Table 7 are:

FOTINI-L, Loaded - 0.602 tons/in.^{2}

FOTINI-L, Ballasted - 0.547 tons/in.^{2}

Figures 17 and 18 could be used in a similar manner with equation (29) to find maximum slamming moments for the E. L. RYERSON and the S. S. MICHIGAN, respectively. The predicted value of a maximum stress calculated using Figures 16 and equation (29) seem to show close enough agreement (with the actual values) to be used in design work. The use of this new method for predicting slam moments and stresses will be discussed next.

Application of the Results

In ship design, the point of interest for bending stress is the deck. This is because the neutral axis is almost always below the mid-depth of the ship, so the section modulus for the deck is usually the mimimum one. Thus, for a given bending moment, the highest stress is in the deck.

The bending stress on a ship in waves can be broken into two components, low frequency hull bending stress and high frequency hull bending stress. These components are caused by low frequency bending moments and high frequency bending moments, respectively. Taking the yield stress as the maximum allowable stress, the following expression is obtained:

$$\sigma_{LF} + \sigma_{HF} \leq \sigma_y \tag{30}$$

where σ_{LF} = low frequency hull bending stress such as is due to the still water and wave bending stresses together

σ_{HF} = high frequency hull bending stress such as is due to slamming or other impact or vibratory loadings

σ_y = yield stress of deck material

Using simple beam theory to solve for σ_{LF} and σ_{HF} and substituting into equation (30) yields,

$$\frac{M_{LF}}{Z} + \frac{M_{HF}}{Z} \leq \sigma_y \tag{31}$$

where M_{LF} = low frequency bending moment

M_{HF} = high frequency bending moment

Z = section modulus of midship section, for the deck

Equation (31) leads to a limiting expression for section modulus which is,

$$Z_{req} \geq \frac{1}{\sigma_y} (M_{LF} + M_{HF}) \tag{32}$$

where the respective bending moments are the maximum values likely to occur simultaneously.

The low frequency moment can be found, for example, by the double integration method of statically balancing the ship on a standard wave. The high frequency moment can then be calculated as a function of hull stiffness by the method presented in the previous section. Then a section modulus requirement is obtainable as a function of hull stiffness. An example will help to illustrate this proposed method. For this example the FOTINI-L will again be used. The effect on slam response due to reducing the ship's bending stiffness is desired. The section modulus of the FOTINI-L, as a result of

existing rules is 158,556 in.^{2} ft.* (from Table 6). First, the stiffness will be reduced to 60%. Using a crude estimate, the wave bending moment can be calculated as follows:

$$M_{LF} = \frac{\Delta L}{35} = \frac{(74,203)(800)}{35}$$

$$M_{LF} = 1,696,068 \text{ tons feet}$$

From the methods described in the previous section, the maximum slamming moment for a stiffness of 60% is 70,500 tons feet. Assuming the yield stress is 32,000 psi and substituting into equation (32) gives,

$$Z_{req} \geq \frac{1}{32,000} (1,696,068 + 70,500) \; 2240$$

$$Z_{req} \geq 123,659.8 \text{ in.}^2 \text{ ft.}$$

At a stiffness of 60%, the section modulus for the FOTINI-L is 95,133.6 in.^{2} ft. (from Table 6), which does not satisfy the requirement.

Now a reduction of bending stiffness to 80% will be looked at. The maximum slamming moment turns out to be 75,000 tons feet. Using the same wave bending moment and yield stress

---

*For a new design, this value of section modulus (for stiffness = 100%) would also be figured from the existing rules.

and again substituting into equation (32) gives,

$$Z_{req} \geq \frac{1}{32,000} (1,696,068 + 75,000)\ 2240$$

$$Z_{req} \geq 123,974.8 \text{ in.}^2 \text{ ft.}$$

At a stiffness of 80%, the section modulus for the FOTINI-L is 126,844.8 in.^{2} ft. The requirement is satisfied and so, for this condition, the section modulus must be greater than 123,974.8 in.^{2} ft. to satisfy strength conditions. (Note that the object is to get the actual section modulus as close to the required section modulus as possible.) Thus, from the slamming response point of view, the bending stiffness of the FOTINI-L could be reduced 80% with no ill effects.* Of course, the effects of springing and propellor vibration would have to be studied and the governing effect would set the limiting section modulus requirement at this reduced stiffness.

---

* Similar calculations done for the FOTINI-L indicate that stiffness of 120% and 140% each give a much greater section modulus than is required by equation (32).

CONCLUSIONS

It is seen from Figures 4, 5, and 6 that slamming moments are greatest for small slam durations. This is reasonable since the largest peak forces are experienced at low slam durations, (Figures 4c, and 4d show that for these large peak slam forces, the moment is maximum). It is also noted that, for different stiffnesses, the curves follow the same trend. Thus, there is a family of stiffness curves for each ship in both the loaded and ballasted condition. Another important point is that the bending moment increases as stiffness increases. This suggest an advantage in decreasing the bending stiffness in ships, but stress must also be considered.

Figures 7, 8, and 9 indicate that although the moment decreases for decreased stiffness, the stress still increases. This is true because the section modulus changes more than the slamming moment, as stiffness varies. Also, as suggested by the bending moment graphs, the highest stresses are experienced at large peak slam forces (see Figures 7c and 7d).

Figures 10, 11, and 12 indicate that equation (27) is valid for slamming moments. Figures 13, 14, and 15 confirm this assumption since, for each slam duration, straight lines approximate the data points. Solving equation (27) for each ship and for all slam durations, and comparing calculated moments with actual moments showed a maximum error of 3% for

all cases and a much smaller error in most cases.

The curves in Figures 16, 17, and 18 can be used, for each respective ship, to predict slamming moments amidships. Comparing results from using these graphs [with equation (29)] with actual values of slamming moments (Tables 3, 4, and 5), show close enough agreement that the empirical formula could be used in ship design.

Finally, it is concluded that the application suggested as a criteria for ships' section moduli is a good one, and could eventually be used as an A.B.S. standard.

SUGGESTIONS FOR FURTHER STUDY

For this study the impulsive load was applied at a set point (different for each ship) along the length of the ship. The effect of applying the slam force at different locations along the bottom should be investigated. For example, taking one ship and finding out how the response changes as the impulse is applied different distances from the bow.

Also, the assumption of a half-sine impulse may not be totally correct. Previous studies have shown that a real slam impulse is more steep-sided. Studies should be made on the responses due to different shaped impulses.

Different values of impulse should also be investigated. According to Roger G. Kline, the slam bending moment response is a linear function of the excitation impulse. Thus, the response to any impulse excitation of the same duration and shape can be obtained by linear extrapolation.

Study should be given to the relationship between slam bending moment (and stress) and peak slam load. This thesis' investigation concentrated on slam duration as the main parameter for slamming. Peak force, however, may turn out to be more important.

Some thought should be given to the form of the empirical formula relating ship stiffness to slamming moment, equation (27). It could also be of the form:

$$\text{Slam Moment} = (EI)^{x} + C$$

where the only variables are x and C. This equation is more difficult to solve for two given points (i.e. two sets of moment and stiffness values), but has only two variables to be used in calculating slamming moments.

It is felt that the suggested method for predicting slamming moments could be applied to other ships. Data points for similar type ships should be added to Figures 16, 17, and 18, respectively. If the points followed the same curves then these graphs could be used in general design work for their respective ship type (i.e. Figures 16 for bulk carriers, etc.).

Finally, work must be done to study springing and propellor-excited vibrations. Their effects, along with slamming response, will eventually set the limits on ship stiffness now being sought.

REFERENCES

1. Sellers, M. L. and Kline, R. G., "Some Aspects of Ship Stiffness," S.N.A.M.E., 1967.

2. "Rules for Building and Classing Steel Vessels," America Bureau of Shipping, 1974.

3. Evans, J. H. and Kline, R. G., "First Progress Report on the Hull Girder Stiffness Criteria," unpublished, May 10, 1975.

4. Kline, R. G. and Daidola, J. C., "Ship Vibration Prediction Methods and Evaluation of Influence of Hull Stiffness Variation on Vibratory Response," Ship Structure Committee Report SSC-249, 1975.

5. St. Denis, M. and Fersht, S. N., "The Effect of Ship Stiffness on the Structural Response of a Cargo Ship to an Impulsive Load," Ship Structure Committee Report SSC-186, September 1968.

6. Kline, R. G. and Clough, R. W., "The Dynamic Response of Ship's Hull as Influenced by Proportions, Arrangement, Loading and Structural Stiffness," S.N.A.M.E., 1967.

7. Mansour, A. E. and d'Oliveira, J. M., "Hull Bending Moment Due to Ship Bottom Slamming in Regular Waves," Journal of Ship Research, June 1975.

8. Leibowitz, R. C., "A Method for Predicting Slamming Forces on and Response of a Ship Hull," DTMB Report 1961, September 1963.

9. Ochi, M. K. and Motter, L. E., "Prediction of Slamming Characteristics and Hull Responses for Ship Design," S.N.A.M.E. Trans., 1973.

10. McGoldrick, R. T., "Ship Vibration," DTMB Report 1451, December 1960.

11. Kaplan, R. and Sargent, T. P., "Further Studies of Computer Simulation of Slamming and Other Wave-Induced Vibratory Structural Loadings on Ships in Waves," Ship Structure Committee Report SSC-231, 1972.

12. Ochi, M. K. and Motter, L. E., "Prediction of Extreme Values of Impact Pressure Associated with Ship Slamming," Journal of Ship Research, June 1969.

13. Little, R. S. and Lewis, E., "A Statistical Study of Wave-Induced Bending Moments on Large Oceangoing Tankers and Bulk Carriers," S.N.A.M.E., 1971.

14. Evans, J. H., "Ship Structural Design Concepts," Cornell Maritime Press, 1975.

APPENDIX

The following pages contain sample graphs output by the Kline-Clough program. These and similar plots were used to find the actual slamming moments for the FOTINI-L, STR. E. L. RYERSON, and S. S. MICHIGAN.

These specific graphs are for the FOTINI-L at a bending stiffness of 80%. The labeling is as follows:

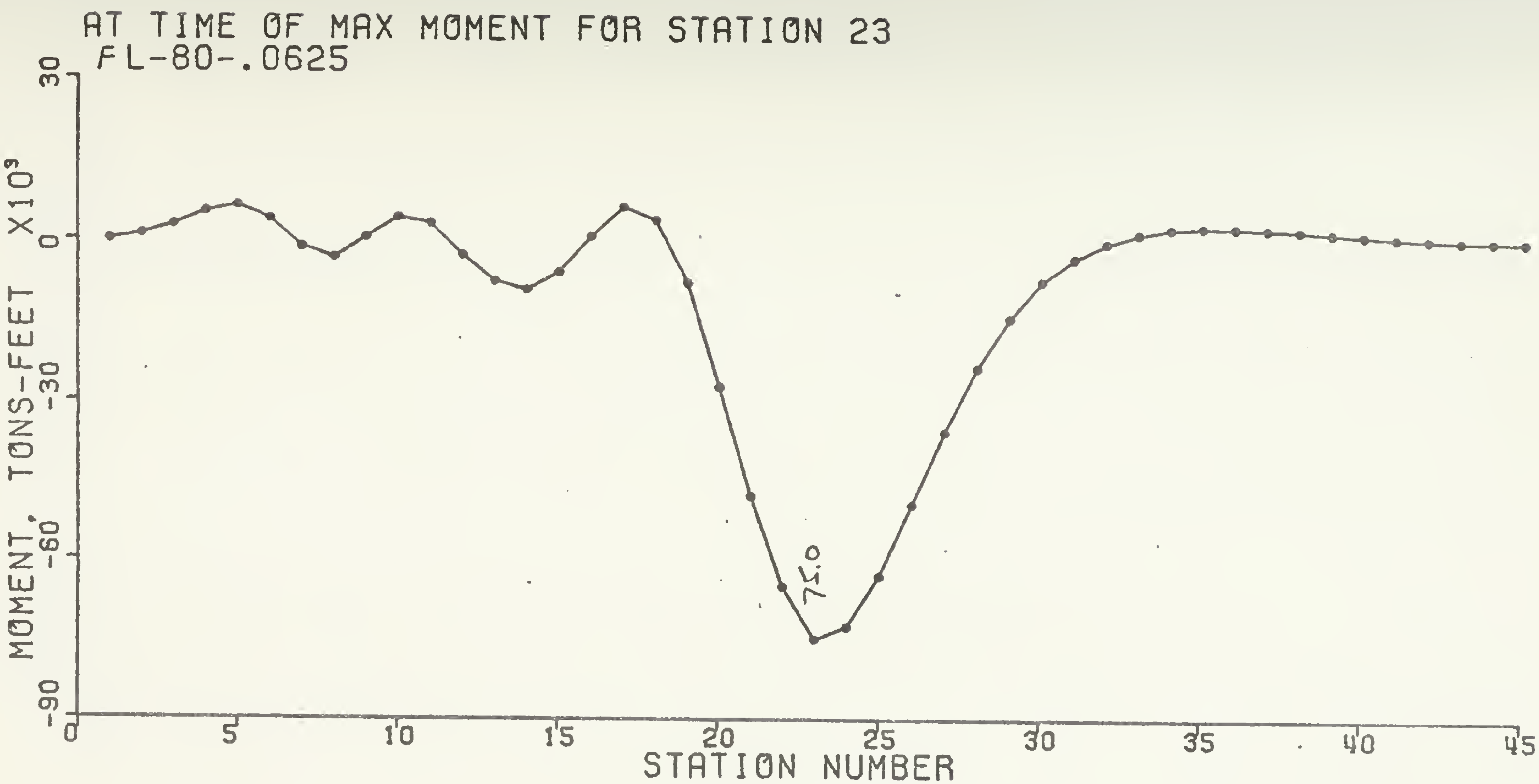
FL = FOTINI-L in loaded condition

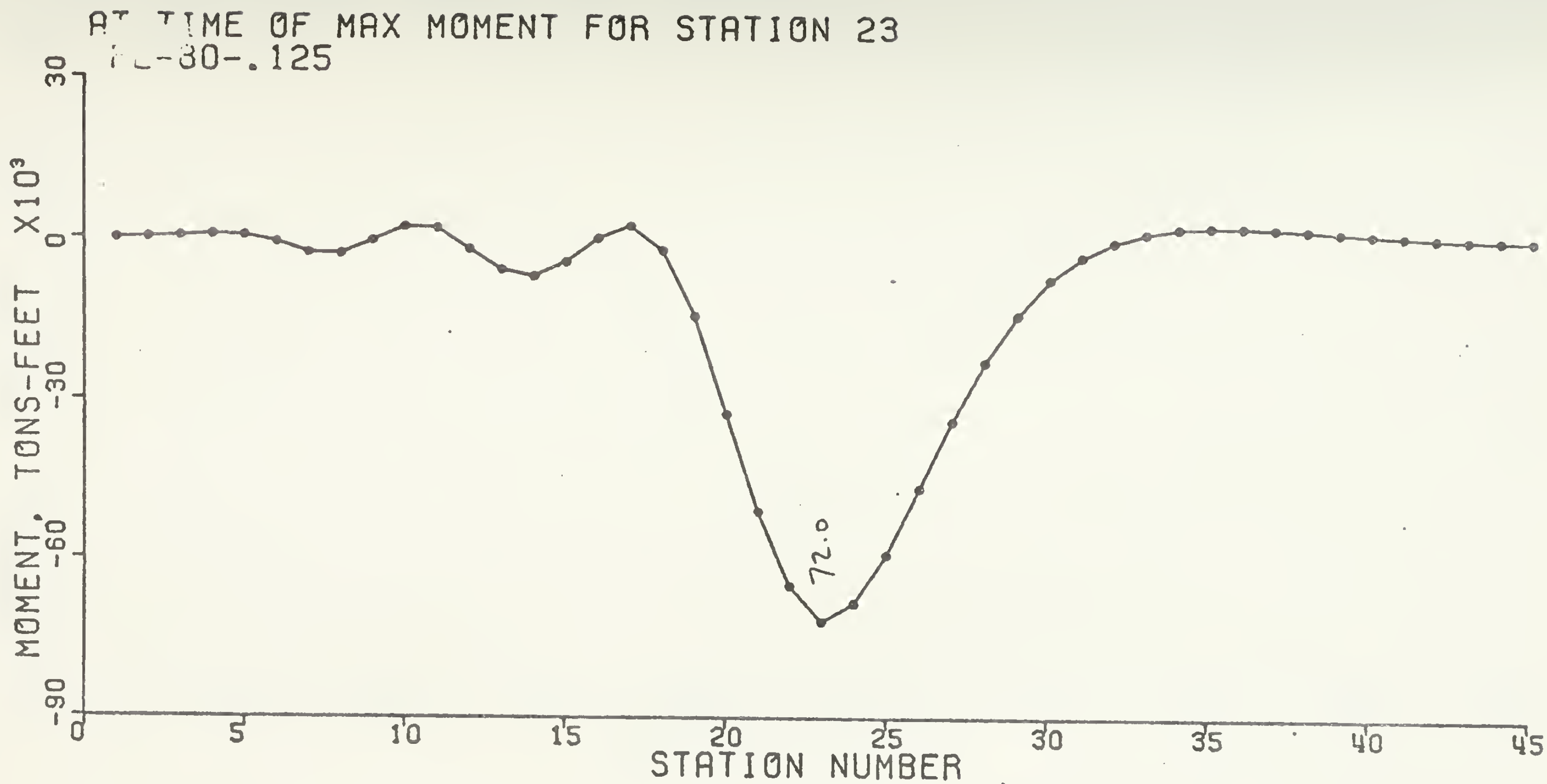
FB = FOTINI-L in ballasted condition

80 = hull bending stiffness

0.0625,
0.125, etc. = slam duration in seconds

Note that the program plots have broken the ship into 44 stations with the F.P. (station 1) to the left.

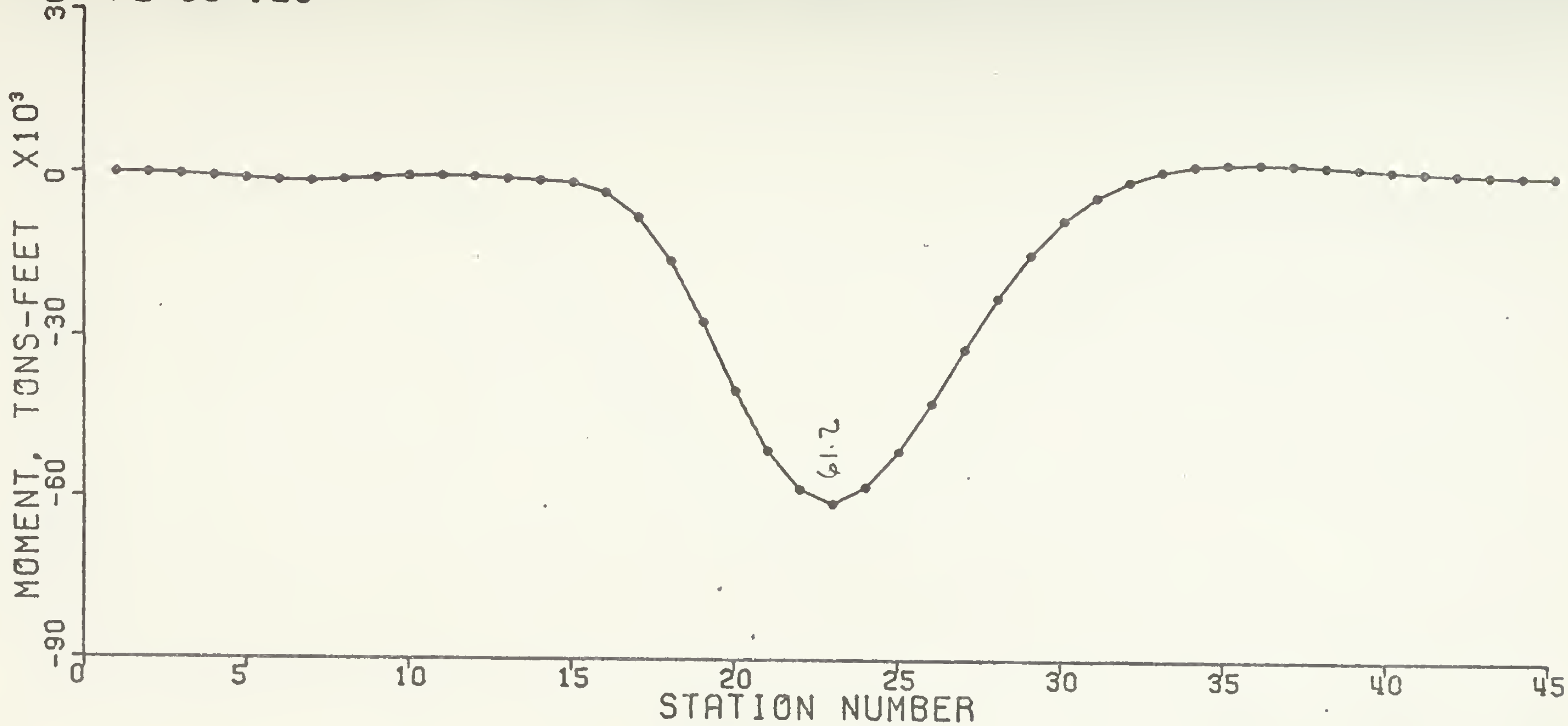

AT TIME OF MAX MOMENT FOR STATION 23
FL-80-.0625
MOMENT, TONS-FEET X10³
30
0
-30
-60
-90
0
5
10
15
20
25
30
35
40
45
STATION NUMBER
75.0



AT TIME OF MAX MOMENT FOR STATION 23
-30-.125
MOMENT, TONS-FEET X10³
30
0
-30
-60
-90
0
5
10
15
20
25
30
35
40
45
STATION NUMBER
72.0

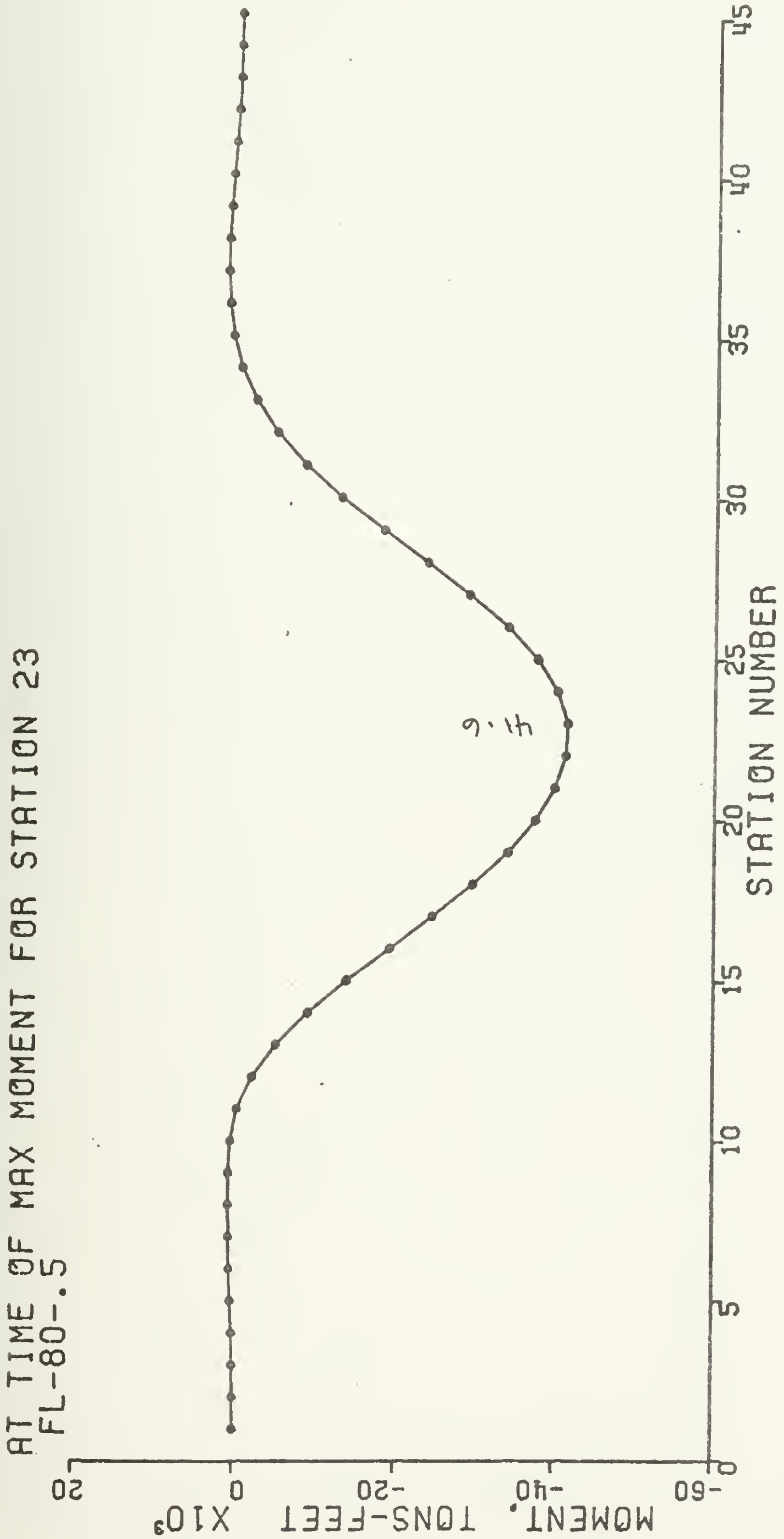
107

AT TIME OF MAX MOMENT FOR STATION 23
FL-80-.25

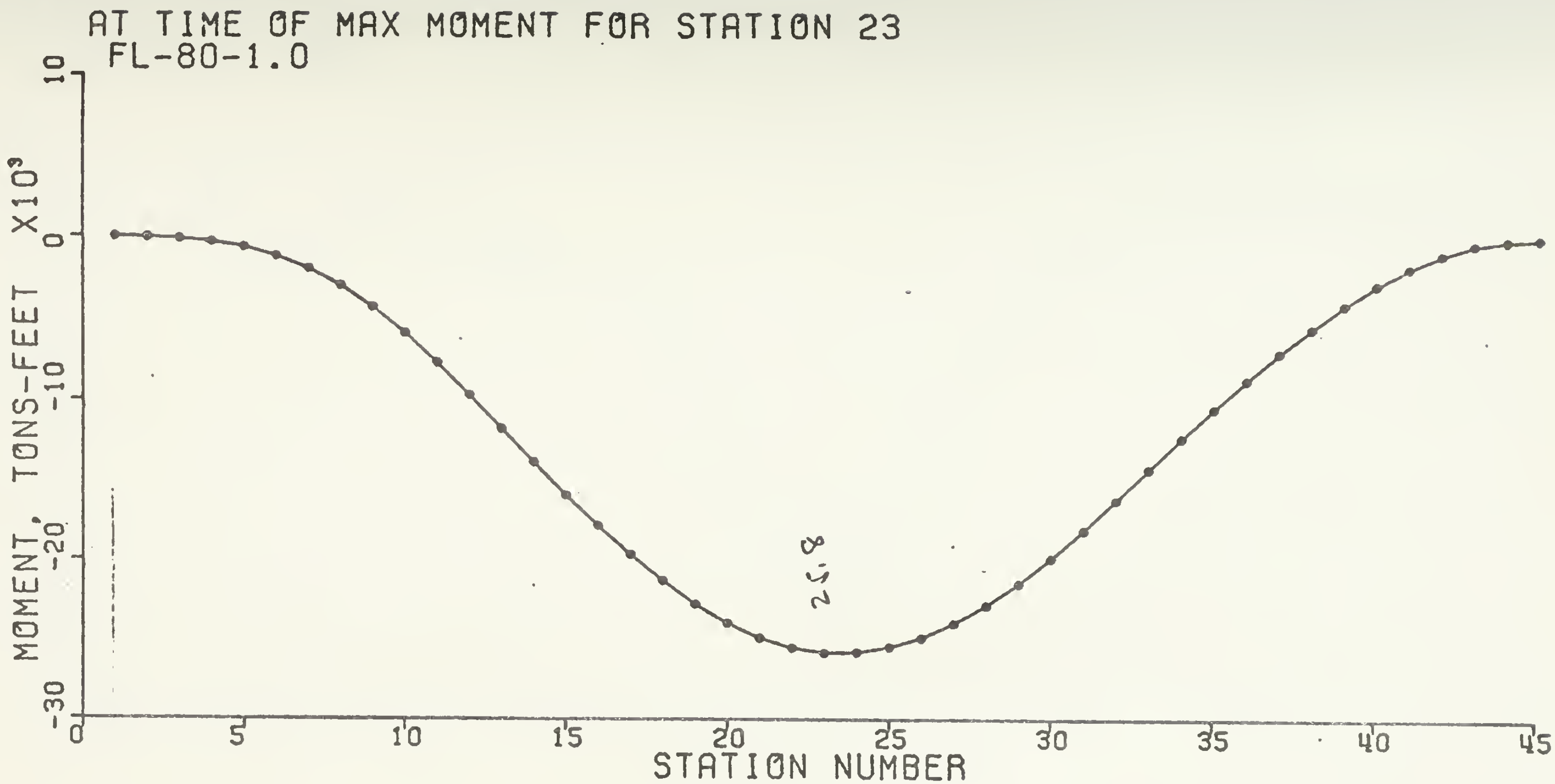


(112)

17

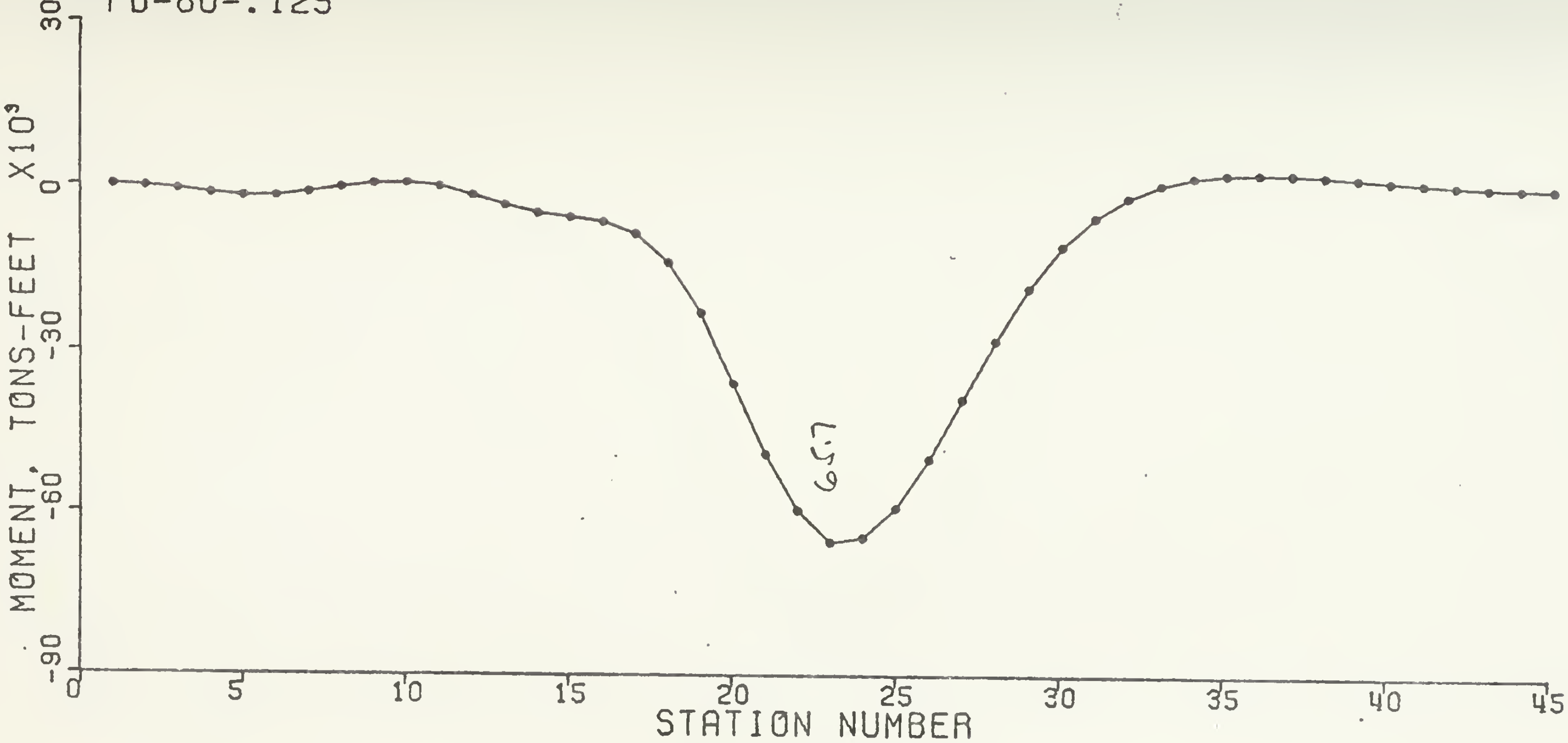


AT TIME OF MAX MOMENT FOR STATION 23
FL-80-.5
STATION NUMBER
MOMENT, TONS-FEET X10³
41.6


AT TIME OF MAX MOMENT FOR STATION 23
FL-80-1.0
10
0
-10
-20
-30
MOMENT, TONS-FEET X10³
0
5
10
15
20
25
30
35
40
45
STATION NUMBER
25.8

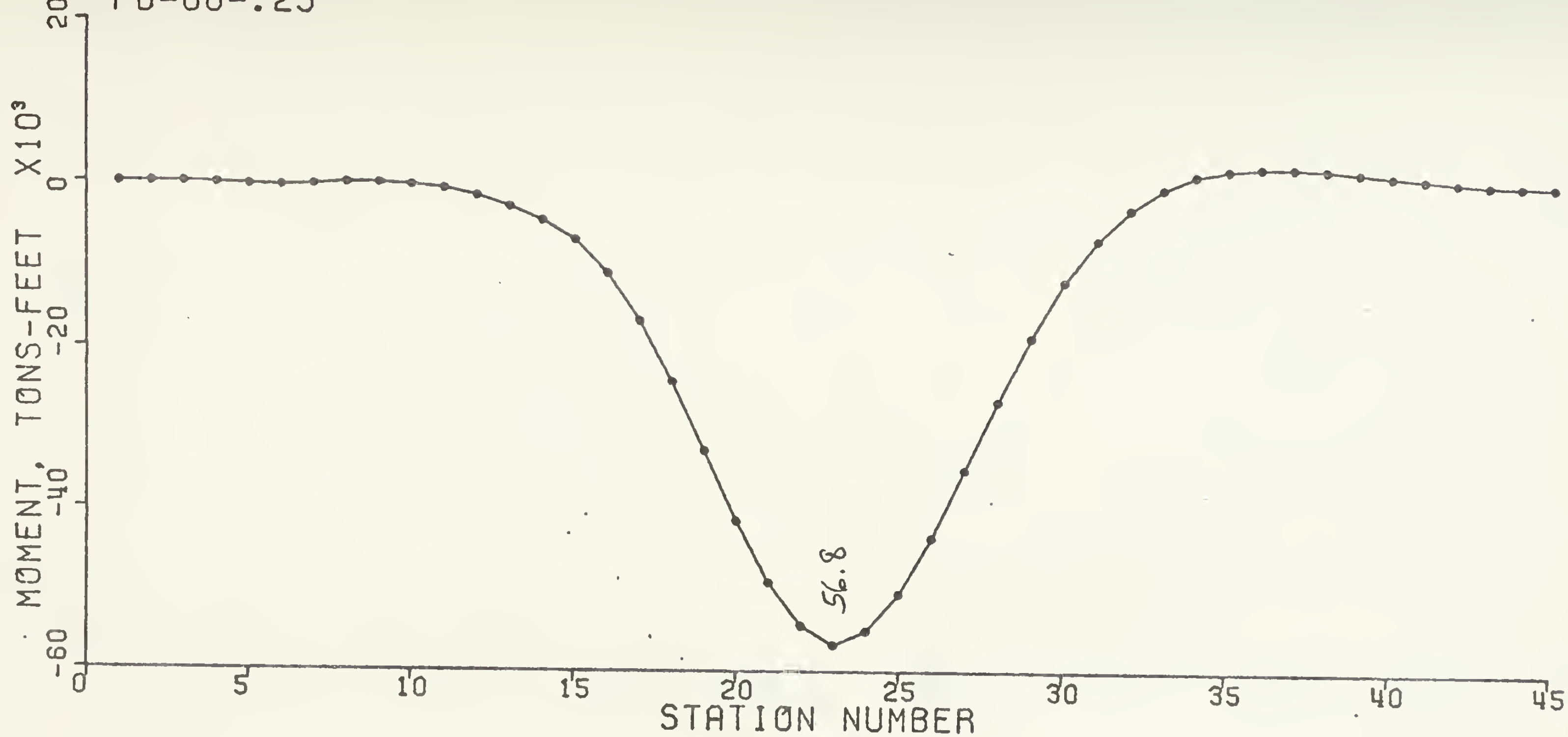

AT TIME OF MAX MOMENT FOR STATION 23
FB-80-.0625
68.7
STATION NUMBER
0
5
10
15
20
25
30
35
40
45
MOMENT, TONS-FEET X10
30
0
-30
-60
-90

AT TIME OF MAX MOMENT FOR STATION 23
FB-80-.125



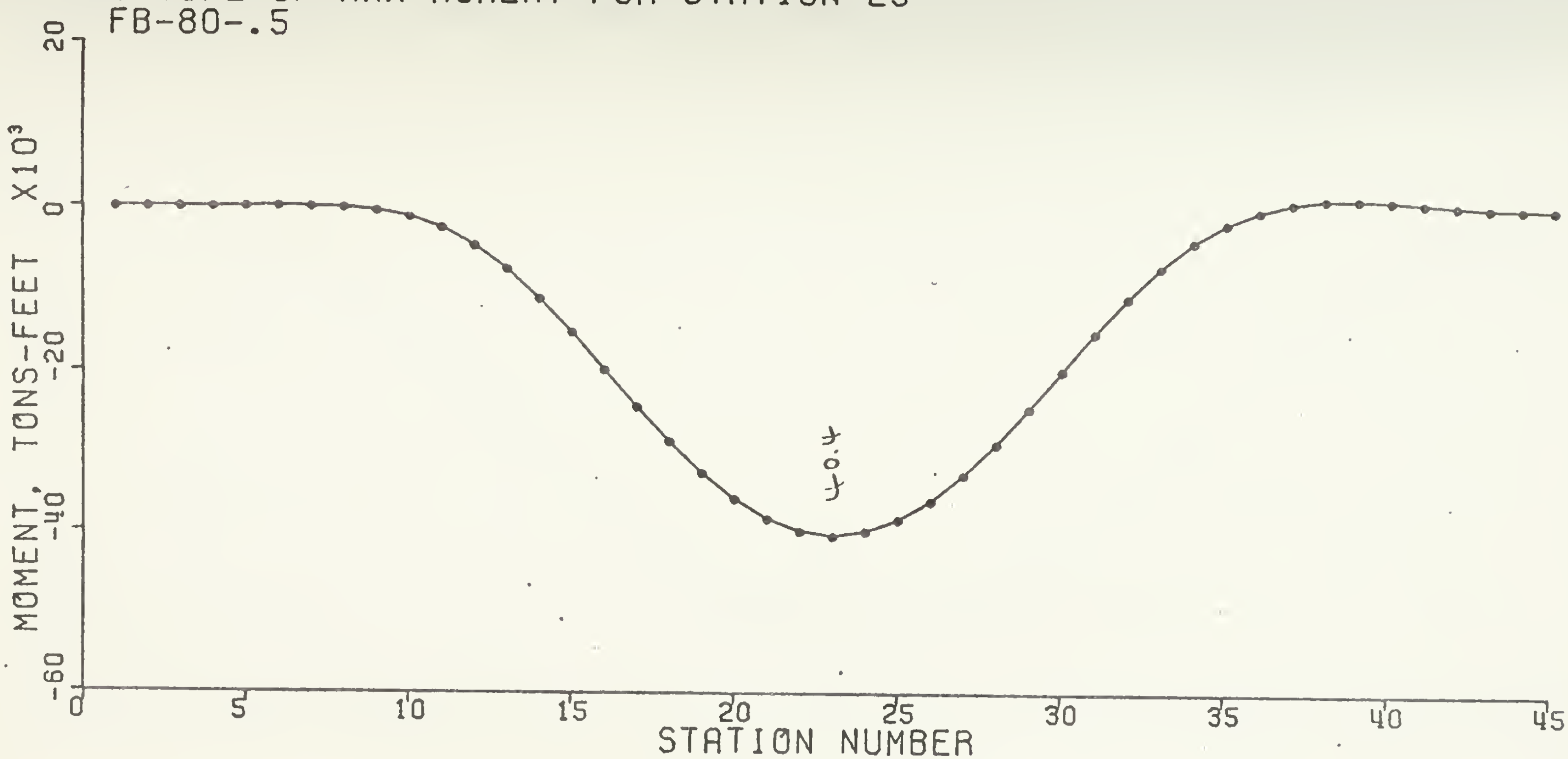
132

AT TIME OF MAX MOMENT FOR STATION 23
FB-80-.25

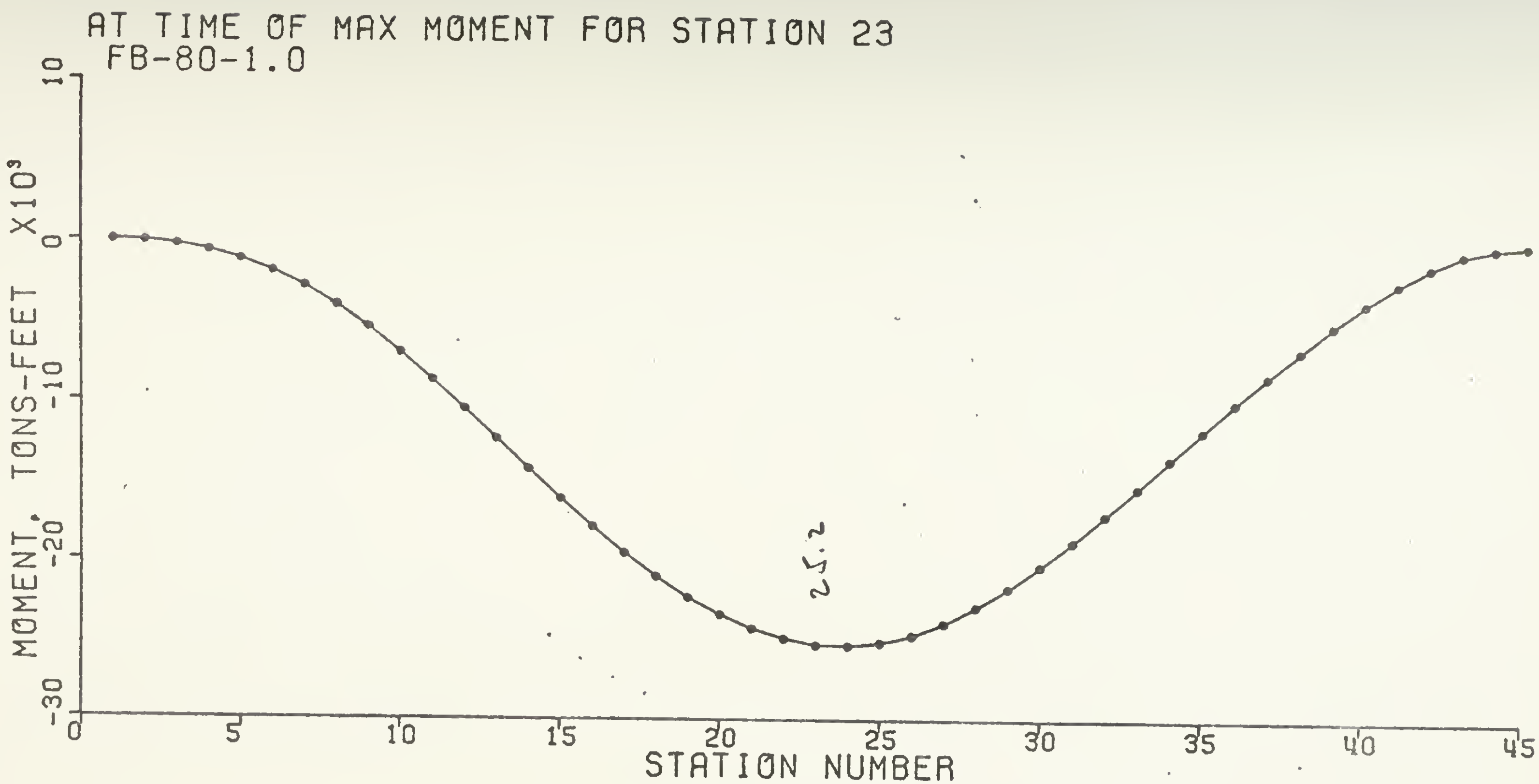


(137)

AT TIME OF MAX MOMENT FOR STATION 23



142


AT TIME OF MAX MOMENT FOR STATION 23
FB-80-1.0
MOMENT, TONS-FEET X10³
10
0
-10
-20
-30
0
5
10
15
20
25
30
35
40
45
STATION NUMBER

147

163791
Thesis
D6483 Donovan
A study of the relation between ship stiffness and maximum slamming moments amidships.